'GHHGEVU'QH'O GEJ CP KECN'RTQRGTVKGU'QP 'Y CXG'RTQRCI CVKQP '' ""'QH'GZVGP FGF 'XKUEQGNCUVKE '"GWNGT/DGTP QWNNKDGCO 'D['" """""WUKP I 'Y CXGNGV'URGEVTCN'HKP KVG'GNGO GP V'O GVJ QF

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1. Introduction

Wave propagation analysis in structures is of much importance for its applications to problems like structural health monitoring (SHM) and non-destructive tests (NDT). Wave propagation problems deal with high frequency excitations and the FE modelling is not computationally viable as the element size has to be comparable to wavelengths, which are very small at higher frequencies. In the past decades, many different approaches have been published for signal processing like Fast Fourier Transform (FFT), Short Time Fourier Transform (STFT) and etc. These approaches mainly based on dividing time domain to constant fix window of time. Wavelet Transform (WT) is an efficient method for variant oscillations functions analysis like wave analysis. The advantage of WT over other methods is the use of wavelet and scaling functions. This makes variable time window due to oscillation of function in special domain, so, in this approach there is no noise in response in ideal condition.

Complex modulus is a property of viscoelastic materials. It is the ratio of stress to strain under vibratory conditions (calculated from data obtained from either free or forced vibration tests, in shear, compression, or tension). Complex variables can be used to express the moduli E^* and G^* follows:

(1)
$$E^* = E' + iE''$$
$$G^* = G' + iG''$$

In present work, wavelet transform and spectral finite element used to temporal approximation and eigenvalue analysis respectively for Euler-Bernoulli beam formulation. Viscoelastic properties applied in the form of complex modulus time dependent properties. Response of beam (transient velocity) under impulse and tone burst loading extracted for SHM algorithm by studying the effect of mechanical properties of material.

2. Extended Euler-Bernoulli Beam equation

The Euler-Bernoulli Beam governing equation as follows:

(2)
$$E^* I \frac{\partial^4 w}{\partial x^4} + \rho A \frac{\partial^2 w}{\partial t^2} = 0, \quad E^* A \frac{\partial^2 u}{\partial x^2} = \rho A \frac{\partial^2 u}{\partial t^2}$$

Wavelet transformed equation is:

(3)
$$E^* \frac{d^2 \hat{u}_j}{dx^2} = -\frac{\rho}{\Delta t^2} \gamma_j^2 \hat{u}_j, \quad E^* I \frac{d^4 \hat{w}_j}{dx^4} = \frac{\rho A}{\Delta t^2} \gamma_j^2 \hat{w}_j$$

Where γ_j^2 is diagonal terms of connection coefficents of Daubechies compactly suppoerted wavelet[1]. Essential boundary condition must be transformed to wavelet space. This helps the model for applying arbitrary loading.
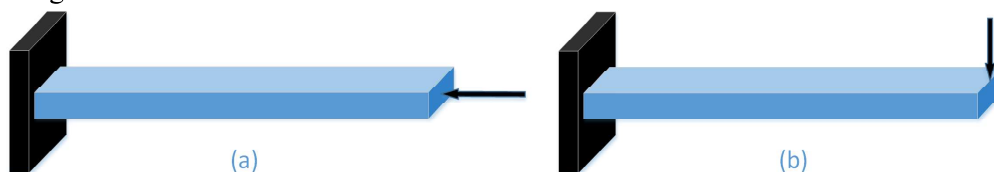

Figure 1.Applied (a)longitudinal (b)transverse loading

3. Problem Solution

Problem is solved by Matlab numerical code. Validation of problem is done with results in [3] with Aluminum material. For present propose PVC, VLDPE and HDPE viscoelastic polymer material properties considered based in [2]. Comparing transient velocity of this material based on properties effect helps to improve SHM algorithm.

4. Results

Response of longitudinal impulse loading applied at the free end by FEM and WSFEM compared in Figure 2. It is well seen that FEM does not provide an accurate response in this case. Comparison of viscoelastic and elastic ($E'' = 0$) of VLDPE behavior under impulse loading is shown in Figure 3.
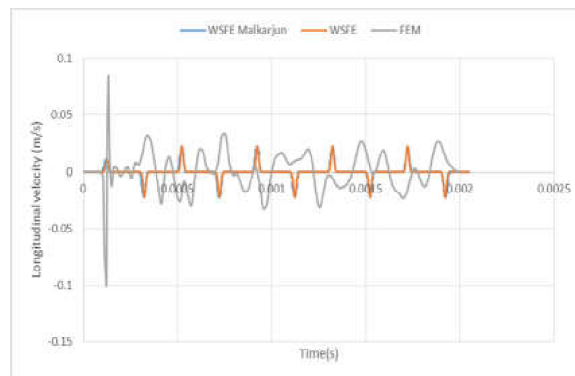

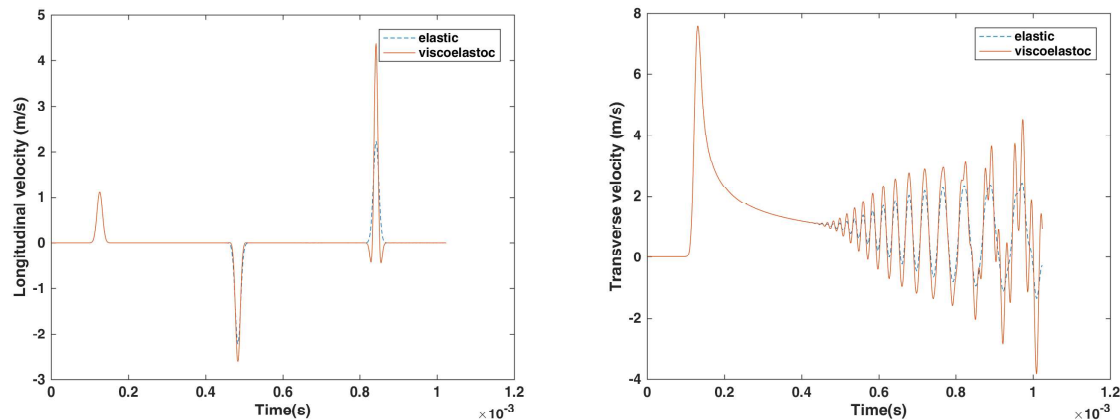*Figure 2. Longitudinal velocity at the free end of beam*

Figure 3. Response of VLDPE beam in elastic and viscoelastic properties under impulse load at the free end of beam

5. Conclusions

Wavelet spectral finite element is an efficient low cost method to wave propagation analysis. Viscoelastic properties of material have different response as compared to elastic properties that is so important because in viscoelastic properties stress may increase by 100%. It means that if only elastic properties are considered in polymer material, it can cause a high risk in design.

References

[1] Gopalakrishnan, Srinivasan; Mitra, Mira. *Wavelet methods for dynamical problems: with application to metallic, composite, and nano-composite structures*. CRC Press, 2010.
[2] Hay, J.; Herbert, E. Measuring the complex modulus of polymers by instrumented indentation testing. *Experimental Techniques*, 2013, 37.3: 55-61.
[3] B. Mallikarjun, "Response of extended Euler-Bernoluli beam under impulse load using wavelet spectral finite element method," National Institute of Technology, Rourkela, 2012.