DISCRETE AND EQUIVALENT 6-PARAMETER SHELL APPROACH TO SIMULATE MECHANICAL BEHAVIOR OF TENSEGRITY LATTICES

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1. Introduction

For the purpose of this paper tensegrities are defined as cable-strut structures consisting of isolated compressed elements inside a continuous net of tensioned members [6]. Node configuration of these structures ensures occurrence of infinitesimal mechanisms that are balanced with self-stress states [1, 4]. Tensegrity lattices are complicated regarding both their geometry and mechanical properties. In order to describe their actual properties and identify features of the structure as a whole, a shell continuum model is considered within the 6-parameter theory. The shell parameters are calibrated with the use of discrete model based on algebraic equations of the problem.

2. Discrete model and 6-parameter shell theory

Discrete model of the structure is composed of *e* straight and prismatic bars of the lengths l_k, cross sections A_k and Young modulus E_k. The bars are connected in nodes in which the number of *s* nodal displacements q_j and nodal forces Q_i are defined [5]. Axial forces N_k can be expressed by the extensions of bars Δ_k in the form $N_k = E_k A_k \Delta_k / l_k$. The extensions Δ_k are a combination of nodal displacements $\Delta_k = \sum_{j=1}^{s} B_{kj} q_j$, $j = 1,2,...,s$. Additionally the self-equilibrated system of axial forces S_k which satisfy the homogeneous set of equilibrium equations $\sum_{k=1}^{e} B_{jk} S_k = 0$ is considered. If one consider equations of equilibrium in the actual configuration then moment $M_k = S_k l_k \psi_k$ is acting on each bar. Angles of bar rotations ψ_k can be expressed as a combination of nodal displacements $\psi_k = \frac{1}{l_k} \sum_{j=1}^{s} C_{kj} q_j$. The above formalism leads to the linear system of algebraic equations $\sum_{j=1}^{s} (k_{ij} + k_{ij}^G) q_j = Q_i$, in which the linear stiffness matrix k_{ij} and geometric stiffness matrix k_{ij}^G can be expressed in algebraic form $k_{ij} = \sum_{k=1}^{e} B_{ki} \frac{E_k A_k}{l_k} B_{kj}$, $k_{ij}^G = \sum_{k=1}^{e} C_{ki} \frac{S_k}{l_k} C_{kj}$ (see [5, 7] for further details). The approach is not dependent on any approximation typical for the finite element method.

Equivalent continuum model of the tensegrity shell-like structure is based on the linearized 6-parameter shell theory [3]. The subject under consideration is a shell of thickness *h*. Displacement field is described by three linear displacements u_α, w of middle surface and three rotations ϕ_α, ψ. Full description of all equations of the theory can be found in [3]. Linear constitutive relations are crucial from the point of view of the continuum model equivalent to the discrete model:

$$N_{\alpha\beta} = B^0_{\alpha\beta\lambda\mu}\,\gamma_{\lambda\mu} + B^1_{\alpha\beta\lambda\mu}\,\kappa_{\lambda\mu},\ M_{\alpha\beta} = \frac{h^2}{12}B^0_{\alpha\beta\lambda\mu}\,\kappa_{\lambda\mu} + B^1_{\alpha\beta\lambda\mu}\,\gamma_{\lambda\mu},$$
$$N_{\alpha 3} = k^2 B^0_{\alpha 3\beta 3}\,\gamma_{\beta 3} + m^2 B^1_{\alpha 3\beta 3}\,\kappa_{\beta 3},\ M_{\alpha 3} = \frac{h^2}{12} l^2 B^0_{\alpha 3\beta 3}\,\kappa_{\beta 3} + m^2 B^1_{\alpha 3\beta 3}\,\gamma_{\beta 3}, \tag{1}$$

where: $\gamma_{\alpha\beta},\ \kappa_{\alpha\beta},\ \gamma_{\alpha 3},\ \kappa_{\alpha 3},\ \gamma_{33}$ – strain components, $N_{\alpha\beta},\ M_{\alpha\beta},\ N_{\alpha 3},\ M_{\alpha 3}$ – internal forces, $k^2,\ l^2,\ m^2$ – correction factors. The relations for tensegrity-like anisotropic shells are introduced below.

3. Mechanical behavior of tensegrity shell-like continuum

The first step of the proposed modelling is selection of a repetitive segment, which is taken out from the tensegrity shell-like structure. Then, the selected representative segment undergoes numerical homogenization [1]. By comparing the elastic strain energy from FEM truss formulation with the energy of a solid, a continuum model of the segment is obtained. The homogeneous segments are afterwards joined together to create a three-dimensional orthotropic continuum, which includes the effect of self-stress [1]. After applying the assumptions of shell theory and integration over the thickness, a two-dimensional shell model is obtained for membrane, bending and transverse shear deformations. The model includes the effect of self-stress that was initially applied to the tensegrity discrete model. For example, the selected coefficient for flat structure composed of regular extended octahedron tensegrity modules connected with additional cables can be expressed as [2]:

$$B^0_{1111} = \frac{2EA}{h}\left(1 + 1{,}52325 \cdot p + 0{,}13125 \cdot n + 0{,}129225 \cdot \sigma\right), \tag{2}$$

with the following connection/strut/cable properties: $n = (EA)_{con} / (EA)_{str}$, $p = (EA)_{cab} / (EA)_{str}$, $(EA)_{str} = EA$ with self-stress multiplier $\sigma = S / EA$. If general shall-like tensegrity lattices are considered the mechanical behavior depends on: tensegrity module used, self-stress applied, geometry of the modules, cable/strut/connection properties and Gaussian curvature of the shell model. A separate problem is to define the values of correction factors $k^2,\ l^2,\ m^2$ for various tensegrity lattices. The above problems will be presented in detail and discussed during the conference.

4. References

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