# MODELLING OF WEDGE INDENTATION USING A GRADIENT-ENHANCED CRYSTAL-PLASTICITY MODEL

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## 1. Introduction

Size effects is metal plasticity attract a significant interest in the materials science and mechanics communities. This work is concerned with the modelling of the size effects induced by strain gradients, for instance, in micro-torsion, micro-bending, and micro/nano-indentation. The related hardening mechanism ('smaller is stronger') is associated with the geometrically necessary dislocations (GNDs) that accommodate the strain gradients [1].

Modelling of the indentation size effect is in most cases limited to isotropic plasticity, often based on a simplified geometric model, e.g. [2]. Simulations employing gradient crystal plasticity models are much more scarce. In this work, a recently developed gradient-enhanced crystal-plasticity model [3,4] is applied to predict the size effects in wedge indentation.

# 2. Minimal gradient enhancement of crystal plasticity

In the model of Petryk and Stupkiewicz [3], the classical framework of crystal plasticity is enhanced with sliprate gradient effects by extending the usual anisotropic hardening law with a single isotropic term that represents the GND hardening. Unlike in the frequently used split of the total dislocation density into the densities of statistically stored dislocations (SSDs) and GNDs, e.g., [1, 2], such a split is applied in an incremental form only. The difference may seem minor, but it has a significant influence on the resulting model. The internal length scale  $\ell$ ,

(1) 
$$\ell = \frac{a^2 \mu^2 b}{2\tau \theta},$$

depends on the current flow stress  $\tau$  and hardening rate  $\theta$ , the remaining parameters being essentially known for a given material. The length scale  $\ell$  has been shown to be closely related to the mean free path of dislocations and thus possesses a direct physical interpretation [3]. The resulting 'minimal' gradient enhancement of the hardening law is thus free of any fitting parameters. Specifically, the following gradient-enhanced hardening law for the critical resolved shear stresses  $\dot{\tau}^c_{\alpha}$  has been derived,

(2) 
$$\dot{\tau}_{\alpha}^{c} = \theta \bigg( \sum_{\beta} q_{\alpha\beta} |\dot{\gamma}_{\beta}| + \ell \dot{\chi} \bigg),$$

where the slip-rate gradient effects are introduced through the term  $\ell \dot{\chi}$ , and  $\dot{\chi}$  is the effective slip-rate gradient. For  $\ell \dot{\chi} = 0$ , the usual hardening law is recovered with  $q_{\alpha\beta}$  denoting the latent-hardening interaction matrix.

## 3. Simulation of wedge indentation into nickel single crystal

The model has been applied to simulate wedge indentation into a nickel single crystal [5]. The experimental results reported by Dahlberg et al. [6] and Sarac et al. [7] have been used for verification of the model in the range of relatively large indentation depths (about  $200 \,\mu$ m) for which the gradient effects are expected to be negligible. A good agreement with the experiment has been obtained in terms of the indentation load–penetration depth curves for three wedge angles, as well as in terms of the distributions of lattice rotation,

GND density, and net Burgers vector. At the same time, the results obtained confirm that the effect of slip-rate gradients is indeed negligible for the indentation depth of  $200 \,\mu$ m. Subsequently, the size effects have been studied by varying the maximum indentation depth in the range between  $200 \,\mu$ m and  $1 \,\mu$ m. In this range of indentation depths, the size effect manifests itself in the increase of hardness by the factor of approximately four with respect to the large penetration-depth limit. This is also accompanied by size-dependence of other features, including the residual imprint. Sample results are shown in Figs. 1 and 2.



Figure 1: Wedge indentation into a nickel single crystal: deformed mesh in the vicinity of the indent (left), indentation load–penetration depth curve (middle), and the dependence of hardness on the maximum indentation depth (right).



Figure 2: Distribution of GNDs near the indent: experiment [7] (left) and simulation (right).

#### References

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