ELASTOSTATIC PROBLEM OF TWO ASYMMETRICAL CRACKS ALONG THE INCLUSION-MATRIX INTERFACE UNDER A REMOTE UNIFORM LOAD

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1. Abstract

Heterogeneous geomaterials as well as composite industrial materials are widely studied because of their applications to engineering problems or use in industrial processes. In these materials, the interface plays a very important role in the mechanical behavior. It ensures the transmission of forces between the matrix and the inclusion during a solicitation. In fact, one of common failure modes in the composites is the rupture at the interface. Interfacial resistance largely influences the final properties of the composite like stiffness, strength and the fracture behavior. For these reasons, the study of the failure mechanism at the interface of composite materials relating to a imperfect inclusion-matrix interface (i.e a partial debonding at the interface) is essential.

This research focuses mainly on the plane problem of a circular disc shape inclusion in an infinite linear elastic matrix with two cracks which can be asymetrical on its boundary created by a far-field stress (see figure 1).

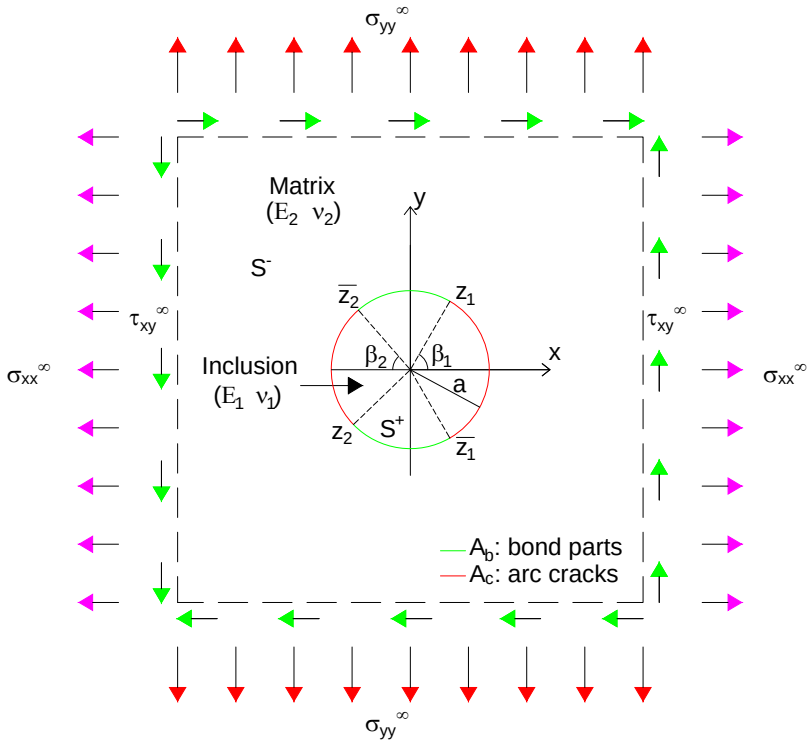

Figure 1: Two asymmetrical cracks at the inclusion-matrix interface under uniform stress at infinity

Among various mathematical methods to solve this problem, we find that the complex potential function method [2] is the more appropriate to take into account the presence of cracks. In this method, stresses and displacements are expressed in terms of analytic functions of complex variables. Significant contributions to this problem have been already reported in the literature. Some authors ([3], [7], [5] and [4]) used this method to study the problem of cracks lying along the interface of a circular inclusion embedded in an infinite solid. However these works have been either limited to a single crack arc or two symmetrical crack arcs. In fact, the

problem of interaction between two asymmetrical cracks at the inclusion-matrix interface is a general solution extending two special cases already studied in literature: A single crack (where one crack has null length) and a two symmetrical cracks (where two cracks have the same length).

The paper starts with the formulation and solution of the problem of two asymmetrical interface arc cracks around a circular elastic inclusion embedded in an infinite matrix subjected to a uniform far load, that is presented in section 2. In section 3, the expression of the complex stress intensity factors (SIFs) at a crack tip along an inclusion-matrix interface is presented. To verify the solution, in section 4, the analytical SIFs for various crack angles in the case of uniaxial tension in plane strain conditions were compared to solution obtained by the Displacement Extrapolation method [1, 6]. The calculation of the total energy release rate (ERR) for the interface crack is carried out in section 5. The conclusions are finally presented in section 6.

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References

[1] Jakob Gager. *Numerical simulations of interface cracks in glued die-attach semiconductor devices*. PhD thesis, TU Wien, 2010.

[2] Nikola Ivanovich Muskhelishvili. *Some basic problems of the mathematical theory of elasticity*. Noordhoff, Groningen, The Netherlands, 1953.

[3] AB Perlman and GC Sih. Elastostatic problems of curvilinear cracks in bonded dissimilar materials. *International Journal of Engineering Science*, 5(11):845–867, 1967.

[4] PBN Prasad and KRY Simha. Interactions of interfacial arc cracks. *International Journal of Fracture*, 117(1):39–62, 2002.

[5] PBN Prasad and KRY Simha. Interface crack around circular inclusion: Sif, kinking, debonding energetics. *Engineering Fracture Mechanics*, 70(2):285–307, 2003.

[6] Yuuki Ryoji and Cho Sang-Bong. Efficient boundary element analysis of stress intensity factors for interface cracks in dissimilar materials. *Engineering Fracture Mechanics*, 34(1):179–188, 1989.

[7] M Toya. A crack along the interface of a circular inclusion embedded in an infinite solid. *Journal of the Mechanics and Physics of Solids*, 22(5):325–348, 1974.