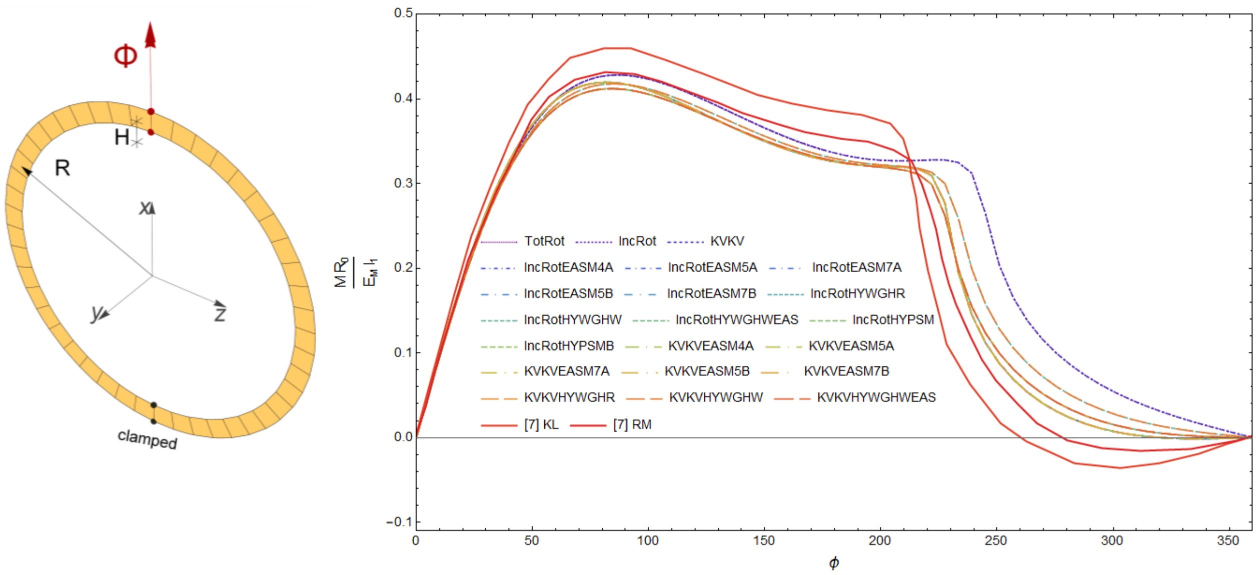
COMPARISON OF SOME LOW-ORDER GEOMETRICALLY EXACT SHELL FINITE ELEMENTS

M. Lavrenčič[1], B. Brank[1]

[1]*Faculty of Civil and Geodetic Engineering, University of Ljubljana, Ljubljana, Slovenia*
e-mail: marko.lavrencic@fgg.uni-lj.si

Description of the work

Low-order (e.g. 4-node) shell finite elements with nonlinear Reissner-Mindlin kinematics (that assumes one inextensible director shell field) are the basic shell finite elements of any commercial finite element code. However, there has been still research going on in order to find an optimal 4-node nonlinear shell finite element of such kind. Mixed variational principles and their modifications have been used for that purpose. In this work, we compare several 4-node large rotations finite element formulations that have been recently published in [2]-[5], [8] and [9]. The formulations are based either on Hellinger-Reissner functional, on Hu-Washizu functional, or on modified versions of Hu-Washizu functional (e.g. ANS (Assumed Natural Strain) and EAS (Enhanced Assumed Strain) concepts) for the membrane, the bending and the shearing parts of the shell response. As for the material models, the St. Venant-Kirchhoff hyperelasticity is considered and the inelastic formulations are under development. Large rotations are described in the same manner for all formulations. The comparison is done by numerical experiments, i.e. by performing an extensive set of standard shell benchmark tests and also some newly proposed tests. Numerical experiments show that some of the formulations are considerable faster than others (since they allow for much larger load increments), and some are more robust. Surprisingly, for some tests, the formulations produce quite different qualitative results for the same mesh.



$R = 20,\ H = \frac{1}{3},$ thickness $th = 1, E = E_M = 21000, \nu = 0.2, \Phi_{final} = 2\,\pi$

Figure 1: Input data and response graph for the “snap-through of an elastic ring” example.

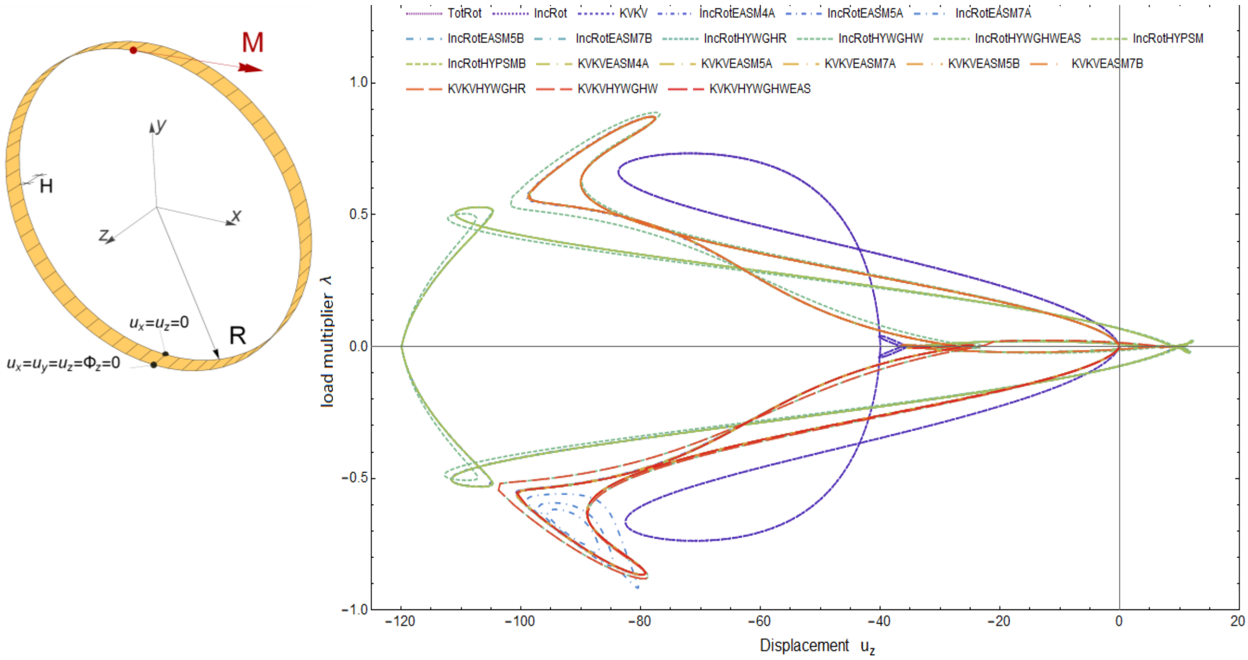
Finite elements

We considered the following large rotation formulations: additive update of the total rotation vector (marked as TotRot in the Figures 1 and 2), additive update of the incremental rotation vector (IncRot), and rotation update by using quaternions (KVKV). As for the 4-node elements, we considered the following formulations. (i) An EAS improvement of the membrane part with 4 (EASM4), 5 (EASM5) and 7 (EASM7) parameters [1], [2]. (ii) Hybrid Hu-Washizu formulation for membrane, bending and shear parts (HYWGHW) [4]. (iii) Hybrid

Hellinger-Reissner formulation for membrane, bending and shear parts (HYWGHR) [3]. (iv) Hybrid Hu-Washizu formulation for membrane, bending and shear parts, enhanced with EAS formulation for the membrane part (HYWGHWEAS) [5]. (v) Hybrid Hellinger-Reissner formulation with Pian-Sumihara interpolation [6] for the membrane part (HYPSM) [6]. (vi) Hybrid Hellinger-Reissner formulation with Pian-Sumihara interpolation for the membrane and bending parts (HYPSMB) [6]. (vii) MITC formulation [9]. For the interpolation of the transverse shear strains, the ANS concept [9] was applied for all formulations.

Results

In Figs. 1 and 2, we show results for two examples. It can be seen, that for these two examples, the elements produce quite different qualitative results for the same mesh.



$R = 30,\ H = 1,$ thickness $th = 1$, $E = 21000$ $\nu = 0.2$, $M = 250$, u_z is displacement of node with applied moment

Figure 2: Input data and response graph for the “jumping rope” example.

References

[1] U. Andelfinger and E. Ramm, EAS-elements for two dimensional, three dimensional, plate and shell structures and their equivalence to HR-elements, *International journal for numerical methods in engineering*, 36: 1311-1337, 1993.

[2] B. Brank, Assessment of 4-node EAS-ANS shell elements for large deformation analysis, *Computational mechanics*, 42: 39-51, 2008.

[3] F. Gruttmann and W. Wagner, A linear quadrilateral shell element with fast stiffness computation, *Computer Methods in Applied Mechanics and Engineering* 194: 4279-4300, 2005.

[4] W. Wagner and F. Gruttmann, A robust non-linear mixed hybrid quadrilateral shell element, *International journal for numerical methods in engineering,* 64: 635-666, 2005.

[5] F. Gruttmann and W. Wagner, Structural analysis of composite laminates using a mixed hybrid shell element, *Comput. Mech.,* 37: 479–497, 2006.

[6] T. H. H. Pian and K. Sumihara, Rational approach for assumed stress finite elements, *International journal for numerical methods in engineering,* 20, 1685-1695, 1984.

[7] B. Oesterle, R. Sachse, E. Ramm and M. Bischoff, Hierarchic isogeometric large rotation shell elements including linearized transverse shear parametrization, *Computer Methods in Applied Mechanics and Engineering,* 321, 383-405, 2017.

[8] K. Wiśniewski, Finite Rotation Shells, Basic Equations and Finite Elements for Reissner Kinematics, *Springer Netherlands*, 2010.

[9] Y. Ko, P.-S. Lee and K.-J. Bathe, The MITC4+ shell element in geometric nonlinear analysis, *Computers and structures*, 185: 1-14, 2017.