MODELLING KINEMATIC HARDENING FOR PROGRESSIVE MEAN STRESS RELAXATION IN PLASTICITY

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Abstract

Several constitutive models for cyclic plasticity have been proposed over the last 40 years. Despite the progress made by most modern approaches, some difficulties still persist, specially regarding a precise descriptions of progressive mean stress relaxation and ratcheting. The present contribution proposes a non standard kinematic hardening model to handle complex loading conditions in low cycle fatigue. The model starts from a non-saturating power law for kinematic hardening in which a relative backstress term is introduced. Such formulation allows a good description of partial mean stress relaxation phenomena under complex loading conditions, at least in proportional cases, with the advantage of need few material parameters. An implicit numerical integration scheme is suggested and implemented to ensure robustness of calculations. The validation is realized considering experimental data available in literature for INCONEL-718DA alloy used in turbine high pressure discs.

1. introduction

The response of continuum damage and other approaches to predict fatigue life are directly dependent on a good description of plasticity phenomena. Nowadays, great efforts have been made to better describe the mean stress relaxation and ratcheting under cyclic multiaxial loadings in inelastic regimes. The mean stress effect in high cycle fatigue (HCF), well known and often satisfactorily represented in uniaxial stress cases by Goodman and Soderberg linear rules, becomes quiet complex in multiaxial solicitations, [8]. Criterion as Sines and Crossland cannot describe the full triaxiality range nor properly handle non-proportional loadings. In case of low cycle fatigue (LCF), even in uniaxial cases, standard models for kinematic hardening like [1] or based in variations of it, [3], predict a complete mean stress relaxation over few cycles. In [7] a model based in the kinematic hardening governed by a non-saturating law introduced by [2]. Such approach considers a calibration of the material parameter that governs the spring back term accordingly with the loading conditions. It provided some good results in uniaxial traction-compression tests predicting the mean stress relaxation progressively, but its extension to complex cases as variable strain ratio or multiaxial still a open question. Thus, the aim of this work is to propose a kinematic hardening formulation capable to better describe stress-strain amplitude in presence of mean stress. An implicit numerical integration scheme is suggested and some preliminary results are present to evaluate the methodology response. The study validation will consider the experimental campaign of [7] for INCONEL 718DA alloy.

2. Kinematic hardening and mean stress relaxation in plasticity

Kinematic hardening represents the phenomenon known as Bauschinger effect, i.e., the yield surface translation as a rigid body in stress space. A number of constitutive models has been proposed to describe ic behaviour under cyclic loading conditions [6], [1] [3], [4] [5], [2]. Considering the recent advances in the elastoplasticiy characterization, the more modern kinematic hardening laws take the general form:

$$\dot{\boldsymbol{X}} = \frac{2}{3}C\dot{\boldsymbol{\varepsilon}}^p - \mathfrak{B}(\boldsymbol{X}, p, \boldsymbol{\sigma})\dot{\mathcal{P}}_k(\boldsymbol{X}, \boldsymbol{\sigma}, \dot{\boldsymbol{\varepsilon}}^p) \tag{1}$$

where C is a matrial parameter and the product $\mathfrak{B}\dot{\mathcal{P}}$ represents the springback term. Generally, the function $\dot{\mathcal{P}}$ has a proportional and linear dependency in relation to plastic strain rate such as $\dot{\mathcal{P}}_k(\boldsymbol{X}, \boldsymbol{\sigma}, a\dot{\varepsilon}) = a\dot{\mathcal{P}}_k(\boldsymbol{X}, \boldsymbol{\sigma}, \dot{\varepsilon}) \quad \forall a >= 0$. In order to achieve the concave shape of the stress strain curve, the tensorial function $\mathfrak{B}$ is commonly assumed heaving the same sign of $\boldsymbol{X}$and $\|\mathfrak{B}\|$ increasing with $\|\boldsymbol{\sigma}\|$.

2.1 . Alternative formulation for kinematic hardening

One considers the original Desmorat's formulation for kinematic hardening, [2], represented by:

$$\dot{\boldsymbol{X}} = \frac{2}{3}C\dot{\boldsymbol{\varepsilon}}^p - \Gamma X_{eq}^{M-2}\boldsymbol{X}\left\langle \dot{X}_{eq}\right\rangle \tag{2}$$

where C is the modulus of kinematic hardening, $\mathit{\Gamma}$ is a material parameter of the springback term , M is an exponent to control the curvature of stress strain curve and X_{eq} is the von Mises norm of the back stress tensor $\boldsymbol{X}$.

To prevent the fast mean stress relaxation, the general idea is to define the instant when load reversals in plasticity, make $\mathfrak{B}\dot{\mathcal{P}}(\boldsymbol{X}_r) = \boldsymbol{0}$ on it, and maintain the general behaviour of a power law ($\|\mathfrak{B}\| \uparrow$ with $\|\boldsymbol{X}\| \uparrow$) at the subsequent instants. That will ensure $\Delta X_{(uncharge)} = \Delta X_{(charge)}$ for any loading, symmetrical or unsymmetrical, although the concavity due to spring back term holds at both situations.

Thus, it is proposed to change the initial condition in equation (2) at the instant $t = t^\star$ when $\dot{\boldsymbol{X}} : \boldsymbol{X} < 0$ from $\boldsymbol{X}(t^\star) = \boldsymbol{X}$ to $\boldsymbol{X}_r(t^\star) = \boldsymbol{X} - \boldsymbol{X}^\star$, where $\boldsymbol{X}^\star$ is the backstress tensor at infinitesimal of time before $\boldsymbol{X}^\star = \boldsymbol{X}(t^\star - \delta)$. Such modification implies that $\mathfrak{B}\dot{\mathcal{P}}(t^\star) = \boldsymbol{0}$, instant when $\dot{\boldsymbol{X}}$ takes the opposite direction of $\boldsymbol{X}$, and will ensure its growing in norm if $\dot{\boldsymbol{X}}$ holds the same direction (increases or decreases monotonically). Let us introduce the term $\boldsymbol{X}_r = \boldsymbol{X} - \boldsymbol{X}^\star$ and call it relative backstress, then substitute it into equation (2):

$$\dot{\boldsymbol{X}} = \frac{2}{3}C\dot{\boldsymbol{\varepsilon}}^p - \mathit{\Gamma}(X_r)_{eq}^{M-2}\left\langle (\dot{X}_r)_{eq}\right\rangle \boldsymbol{X}_r \tag{3}$$

After that, is necessary to stablish a precise (mathematical) definition of the reversion meaning in plastic strain rate and some important quantities to extend such ideas to multiaxial cases. More detailed informations about the model will be present at final version of the article.

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