SECOND-ORDER TWO-TEMPERATURE MODEL FOR THIN METAL FILM SUBJECTED TO THE ULTRASHORT LASER PULSE

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1. Introduction

Thin metal film subjected to the ultra-short laser pulse is considered. Mathematical description of the process discussed is based on the system of four equations. Two of them describe the electrons and lattice temperature, while third and fourth equations represent the generalized Fourier law, it means the dependences between the electrons (lattice) heat flux and electrons (lattice) temperature gradient. Depending on the order of the generalized Fourier law expansion into the Taylor series, the first- and the second-order model can be obtained. In contrast to the commonly used first-order model, here the second-order two-temperature model is considered. The problem is solved using the explicit scheme of the finite difference method. The example of computations is also presented.

2. Governing equations

The two-temperature model describes the temporal and spatial evolution of the lattice and electrons temperatures \( T_l \) and \( T_e \) in the irradiated metal by two coupled nonlinear differential equations [1, 2] (1D problem)

\[
C_e(T_e) \left( \frac{\partial T_e}{\partial t} \right) = -\frac{1}{\partial x} \frac{\partial q_e(x, t)}{\partial x} - G(T_e) \left[ T_e(x, t) - T_l(x, t) \right] + Q(x, t)
\]

and

\[
C_l(T_l) \left( \frac{\partial T_l}{\partial t} \right) = -\frac{1}{\partial x} \frac{\partial q_l(x, t)}{\partial x} + G(T_l) \left[ T_e(x, t) - T_l(x, t) \right]
\]

where \( T_e(x, t), T_l(x, t) \) are the temperatures of electrons and lattice, respectively, \( C_e(T_e), C_l(T_l) \) are the volumetric specific heats, \( G(T_e) \) is the electron-phonon coupling factor which characterizes the energy exchange between electrons and phonons [3], \( Q(x, t) \) the source function associated with the irradiation.

In a place of the classical Fourier law the following formulas are introduced

\[
q_e(x, t + \tau_e) = -\lambda_e(T_e, T_l) \frac{\partial T_e(x, t)}{\partial x}
\]

and

\[
q_l(x, t + \tau_l) = -\lambda_l(T_l) \frac{\partial T_l(x, t)}{\partial x}
\]

where \( \lambda_e(T_e, T_l), \lambda_l(T_l) \) are the thermal conductivities of electrons and lattice, respectively, \( \tau_e \) is the relaxation time of free electrons in metals, \( \tau_l \) is the relaxation time in phonon collisions.

Using the second order Taylor expansion, the equations (3) and (4) can be written in the form

\[
q_e(x, t) + \tau_e \frac{\partial q_e(x, t)}{\partial t} + \frac{\tau_e^2}{2} \frac{\partial^2 q_e(x, t)}{\partial t^2} = -\lambda_e(T_e, T_l) \frac{\partial T_e(x, t)}{\partial x}
\]

and

\[
q_l(x, t) + \tau_l \frac{\partial q_l(x, t)}{\partial t} + \frac{\tau_l^2}{2} \frac{\partial^2 q_l(x, t)}{\partial t^2} = -\lambda_l(T_l) \frac{\partial T_l(x, t)}{\partial x}
\]

The source function \( Q(x, t) \) is associated with the laser irradiation [3]

\[
Q(x, t) = \sqrt{\frac{\beta}{\pi}} \frac{1 - R}{t_p \delta} I_0 \exp \left[ -\frac{x}{\delta} - \beta \frac{(t - 2t_p)^2}{t_p^2} \right]
\]
where $I_0$ is the laser intensity, $t_p$ is the characteristic time of laser pulse, $\delta$ is the optical penetration depth, $R$ is the reflectivity of the irradiated surface and $\beta = 4 \ln 2$.

For $x = 0$ and $x = L$ the non-flux conditions are assumed. The initial condition $T_e(x, 0) = T_l(x, 0) = T_p$ is also known.

### 3. Example of computations

The problem formulated is solved using the finite difference method with staggered grid (Figure 1). As an example of computations, the thin metal films of thickness 100 nm made of gold and copper are considered. The values of thermophysical parameters are taken from [1-4]. The laser parameters are the following: $I_0 = 10 \ J/m^2$ and $t_p = 0.1 \ ps$. In Figure 1 the electrons and lattice temperature histories at the irradiated surface are presented.

![Fig. 1. Discretization](image)

![Fig. 1. Temperature history on the irradiated surface for Au and Cu: a) electrons, b) lattice](image)

### 4. Conclusions

The second-order two-temperature model is considered. The comparison of the results obtained using the first- and second-order models will be shown in the full version of the paper. On this basis, the conclusions will be formulated, in particular for which laser parameters the first-order model is sufficient, and when the higher-order model should be used.

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**References**


