IDENTIFICATION OF A LOAD MOVING ON A PLATE USING THE ℓ_1 NORM MINIMIZATION

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1. Introduction

There are two fundamental inverse problems in the field of structural health monitoring (SHM): identification of damages and identification of loads. Effectiveness of the related computational methods is crucial for maintaining integrity of the monitored structures. This contribution considers identification of unknown loads based on measurements of structural response. It is a relatively extensively researched problem: reviews of techniques used for off-line load identification can be found in [1,2], while techniques for online identification are reviewed in [3].

If the aim is to identify independent force histories in each of the excited degrees of freedom (Dofs), the uniqueness of the solution can be possible only if there are at least as many sensors (equations) as the excited Dofs (unknowns). Such a requirement can be satisfied in case of a few unknown stationary loads, but it becomes problematic if the unknown load is (even single but) moving in an unknown way across the structure. In such a case, a very large number of Dofs can be potentially excited and a limited number of sensors are available to measure the response. As a result, the naïve direct formulation of the inverse problem is underdetermined, and the solution is not unique.

This contribution is devoted to indirect identification of a single moving load that excites a 2D structure (plate). To attain the uniqueness, the solution space needs to be significantly constrained. However, instead of assuming a known trajectory of the load and identifying its value, the aim is to identify the trajectory only. Such a problem is important, e.g., in traffic monitoring and control [4,5]. Effectively, the approach is based on the assumption of sparsity of the excitation, which seems to suit the practice: even if the location of the load is unknown, at each time instant only a single (or a limited number of) Dofs is excited. Such an approach follows the methodology of compressed sensing [6], which includes such SHM-related applications as identification of impact load position [7]. The assumption of sparsity is usually expressed as a requirement of a bounded l1 norm of the solution [8].

The approach has already been verified numerically and experimentally using a flexible 1D structure (a beam) excited with a moving mass [9]. The cases considered there included single or multiple passes of the mass across the beam. The assumption of sparsity allowed the space-time trajectory of the load to be identified. Here, the goal is to test the approach in a much more complex problem that involves a 2D structure, e.g., a plate, subjected to a single moving load. In the fully dynamic case the task is computationally very demanding, thus we focus here on the quasi-static case. This abstract describes briefly the method and the experimental stand. Detailed results will be presented during the conference.

2. Load identification based on sparsity

The considered plate structure is assumed to respond in the linear range. In the quasi-static analysis, the response is decoupled with respect to time and can be expressed as

(1) $$\mathbf{e}_t = \mathbf{B}\mathbf{x}_t,$$

where t indexes the time instants, the vector $\mathbf{e}_t$ collects the structural response measured in time t, $\mathbf{B}$ is the compliance matrix, and $\mathbf{x}_t$ collects the excitations in time t in all Dofs exposed to the moving load. If the length of the unknown excitation vector $\mathbf{x}_t$ is larger than the length of the measurement vector $\mathbf{e}_t$, there are infinitely many solutions. To obtain a unique solution, an additional knowledge about the load has to be used to constrain the solution space. Here, we use the assumption of sparsity, which is expressed through the l_1 norm as the task of minimization of the following weighted objective function:

$$F_t(\mathbf{x}) = \|\mathbf{e}_t - \mathbf{B}\mathbf{x}\|^2 + \alpha\|\mathbf{x}\|_\mathbf{1}, \tag{2}$$

subject to the natural nonnegativity constraints $\mathbf{x} \geq \mathbf{0}$. Notice that (1) can be minimized independently for each time instant t. Alternatively, any desired time-dependent load evolution pattern can be promoted by performing a single minimization of a weighted sum of $\sum_t F_t(\mathbf{x}_t)$ and a term that penalizes the deviations from the pattern.

3. Experimental stand

The experimental stand is designed for identification of a mass moving in a 2D space. The main part is a steel plate (0,0005m x 0.97m x 1m), see Fig. 1. One edge is fully fixed and there are 6 supporting points which restrain the plate in the vertical direction and which are realized by a set screw with a tapered end. The plate is loaded in a vertical direction by the mass of a line follower robot. The robot follows the black thick line painted on the top of the plate. The blue thin lines generate a grid that defines the load identification points. There are 81 equally spaced points. In order to measure the plate response, 7 strain gauges are installed on the bottom side of the plate. The directions and pattern is shown in the draft. There are 6 single strain gauges and one 2-element rosette with 90º stacked grid layout. Detailed results will be presented during the conference.

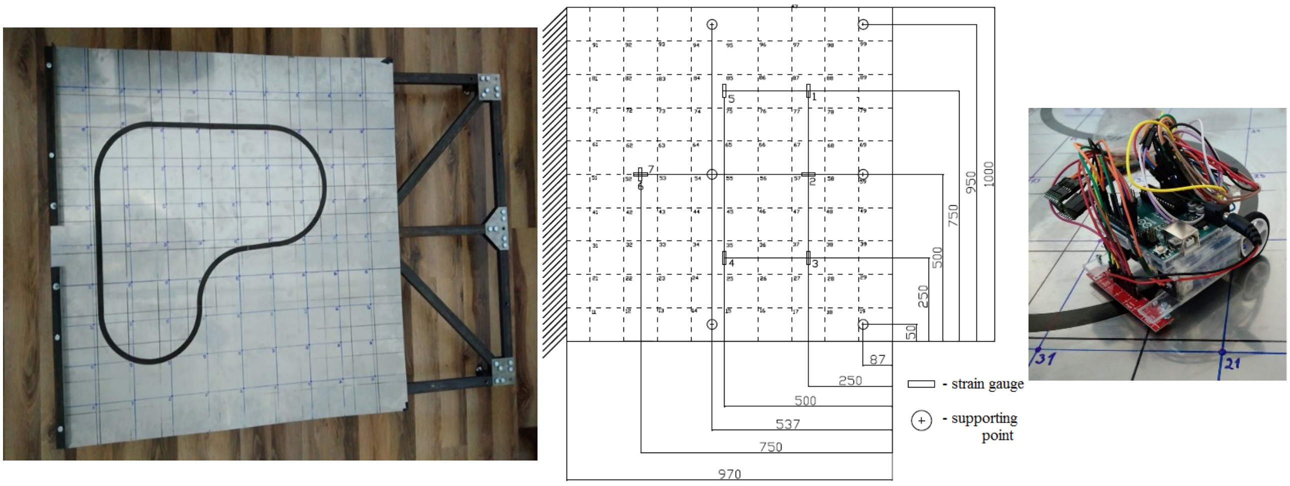

Figure 1: Photographs and a draft of the experimental stand.

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