NUMERICAL MODELLING OF HEAT TRANSFER IN BIOLOGICAL TISSUE DOMAIN USING INTERVAL ANALYSIS

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1. Introduction

In the paper, the numerical analysis of heat transfer process proceeding in the non-homogeneous biological tissue domain subjected to external heat source is presented. In particular, the two-dimensional (axially symmetrical) model with uncertainly defined parameters is considered. The base of mathematical model is given by a set of the Pennes interval equations supplemented by the boundary and initial conditions. The problem discussed has been solved using the interval finite difference method with the rules of directed interval arithmetic [1]. A similar analysis has been done using extended gradual interval arithmetic as the combination of the gradual numbers and Kaucher arithmetic where subtraction and division operators are respectively the inverse operators of the addition and the multiplication [2]. In the final part of the paper the numerical computations obtained for both methods are shown.

2. Interval governing equations

Thermal processes proceeding in the axially symmetrical heterogeneous skin tissue domain can be described by the following system of interval energy equations

$$\left[c_e^-, c_e^+\right]\frac{\partial T_e(r,z,t)}{\partial t} = \left[\lambda_e^-, \lambda_e^+\right]\nabla^2 T_e(r,z,t) + \left[Q_e^-(r,z,t), Q_e^+(r,z,t)\right] \tag{1}$$

where $e = 1, 2, 3$ corresponds to the successive layers of skin (epidermis, dermis, subcutaneous region), $\left[\lambda_e^-, \lambda_e^+\right]$ is the interval thermal conductivity, $\left[c_e^-, c_e^+\right]$ is the interval volumetric specific heat, $\left[Q_e^-(x,t), Q_e^+(x,t)\right]$ is the capacity of interval internal heat sources, $T(r, z, t)$, t is the time, r and z denote the cylindrical coordinates. The interval capacity of internal heat sources is a sum of two components

$$\left[Q_e^-(x,t), Q_e^+(x,t)\right] = \left[G_{Be}^-, G_{Be}^+\right]c_{Be}\left[T_B - T_e(x,t)\right] + \left[Q_{me}^-, Q_{me}^+\right] \tag{2}$$

where $\left[G_{Be}^-, G_{Be}^+\right]$ is the interval perfusion coefficient, c_B is the volumetric specific heat of blood, T_B is the arterial blood temperature, $\left[Q_{me}^-, Q_{me}^+\right]$ is the interval metabolic heat source.

The mathematical model should be supplemented by the boundary and initial conditions.

The skin surface is subjected is subjected to an external heat source assumed in the form (see Figure 1)

$$t \le t_p: \quad q_b(r,0,t) = q_0 \exp\left[-\frac{r^2}{2(R/3)^2}\right] \tag{3}$$

where t_p is the exposure time, q_0 is heat flux corresponding to $r = 0$ and R is the radius of the cylinder. For the others parts of the boundary the no-flux conditions are taken into account (the significant dimensions of R and Z allow to consider such a condition). The initial tissue temperature is also known.

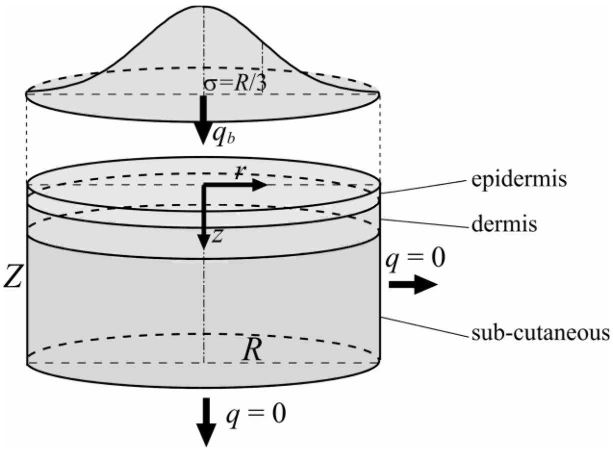

Fig. 1. Domain considered

3. Results of computations

At the stage of numerical computations a three-layered cylindrical skin tissue domain of dimension Z = 12.1 mm and R = 20 mm has been considered. In the first numerical example the thermophysical parameters $\bar{\lambda}_e = \left[\lambda_e - 0.05\lambda_e,\, \lambda_e + 0.05\lambda_e\right]$ and $\bar{c}_e = \left[c_e - 0.05c_e,\, c_e + 0.05c_e\right]$ have been assumed as interval numbers (for e = 1, 2, 3), the time of external heat source exposition is 5.21 s. The other input data are taken from [1]. The numerical model of the problem discussed is based on the finite differential method in the version adapted for the uncertain thermophysical parametrs. All numerical computations has been made using the rules of interval arithmetic, of course. Figure 2 illustrates the heating and cooling curves at the nodes 1 $\left(L_1, 0\right)$, 2 $\left(L_1, r/4\right)$ and 3 $\left(L_1, r/2\right)$ for $q_0 = 5 \cdot 10^3$ W/m^2. The results obtained have been compared to the results of the second example, where the theory of the extended gradual interval arithmetic has been applied. In the full version of the paper, the details of numerical algorithm and also the comparison and discussion of the results obtained will be presented.

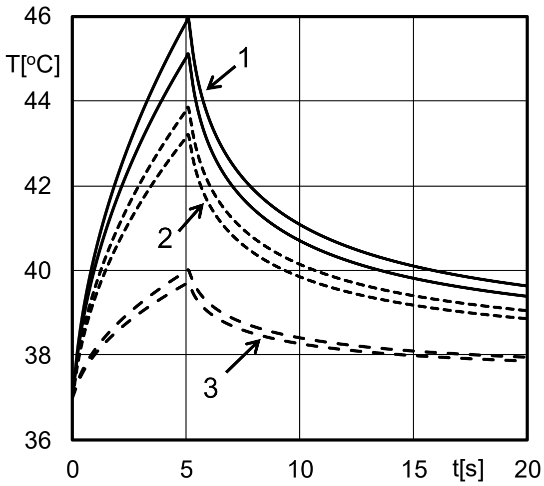

Fig. 2. Solution for directed interval arithmetic.

References

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