

ON NON-LINEAR RESULTANT THEORY OF SHELLS ACCOUNTING FOR THERMODIFFUSION

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1. Introduction

Many different approaches to formulate thermodiffusion theories in 3D bodies have been developed in the literature, see the historical review [4]. In engineering applications to shell structures one needs a 2D model of thermodiffusion based on laws of 3D continuum thermodynamics [5, 7]. Among many important phenomena in shells to be described by such a 2D model is, for example, the transient hydrogen and heat diffusion. It embrittles the materials of shell structures used for hydrogen production, storage and distribution and significantly affects the behaviour of the structures. Another example is the moisture sorption in shell structures made of polymeric materials and composites which may considerably change the prediction of moisture transport processes in shells.

To formulate the 2D model of shell thermodiffusion we apply the direct through-the-thickness integration [1–3] of 3D laws of continuum thermodiffusion [5, 7].

2. Basic 3D relations of non-linear thermodiffusion in solids

Let P with boundary ∂P be a part of the body B in the reference placement. Then thermodiffusion phenomena in 3D bodies are governed by five integral laws of continuum thermodiffusion, which are some extension of those given in [5, 7]. These laws are: balances of body mass, of mass of diffusing medium, of linear and angular momenta, and of energy as well as the entropy imbalance. When the mass production within the body and all inertia effects are disregarded, and then influence of diffusing medium is properly accounted for, these laws in the Lagrangian description relative to the reference placement are:

$$(1) \quad \frac{d}{dt} \iiint_P \rho \, dv = 0, \quad \frac{d}{dt} \iiint_P c \, dv + \iint_{\partial P} \mathbf{n} \cdot \mathbf{j} \, da = 0,$$

$$(2) \quad \iiint_P \rho \mathbf{b} \, dv + \iint_{\partial P} \mathbf{P} \mathbf{n} \, da = \mathbf{0}, \quad \iiint_P \mathbf{y} \times \rho \mathbf{b} \, dv + \iint_{\partial P} \mathbf{y} \times \mathbf{P} \mathbf{n} \, da = \mathbf{0},$$

$$(3) \quad \frac{d}{dt} \iiint_P \rho \epsilon \, dv = - \iint_{\partial P} \mathbf{n} \cdot \mathbf{q} \, da + \iiint_P (\rho r + \mathbf{P} \bullet \text{Grady}) \, dv,$$

$$(4) \quad \frac{d}{dt} \iiint_P \rho \eta \, dv \geq - \iint_{\partial P} \frac{1}{\theta} \mathbf{n} \cdot \mathbf{h} \, da + \iiint_P \frac{\rho r}{\theta} \, dv.$$

In the 3D laws given above, ρ and c are the referential mass (densities) of the body and of diffusing medium, \mathbf{b} , ϵ , r and η are the body force, the internal strain energy, the heat supply and the entropy, all per unit mass of B , \mathbf{P} is the Piola stress tensor field, \mathbf{n} is the exterior unit normal vector of ∂P , \mathbf{y} is the position vector of deformed body relative to an inertial frame, while \mathbf{j} , \mathbf{q} and \mathbf{h} are the fluxes of diffusing medium, of contact heat and of entropy through the boundary ∂P , respectively.

With standard assumptions from (1)–(4) we get the local Lagrangian balance relations. In particular, the local

diffusion equation takes the form

$$(5) \quad \frac{dc}{dt} + \text{Div} \mathbf{j} = 0.$$

Hereinafter Grad and Div are Lagrangian gradient and divergence operators, respectively.

In what follows we are restricting ourselves by nonlinear thermoelastic solids. We introduce the free energy density $\psi = \epsilon - \theta\eta$ which has the form $\psi = \psi(\text{Grad} \mathbf{y}, \theta, c)$. In addition we represent the energy flux \mathbf{q} as follows $\mathbf{q} = \mathbf{h} + \mu \mathbf{j}$, where $\mu = \frac{\partial \psi}{\partial c}$ is the chemical potential. As a result, the global Clausius-Duhem inequality (4) transforms into the local inequality

$$(\text{Grad} \mu) \cdot \mathbf{j} + \frac{1}{\theta^2} \mathbf{h} \cdot \text{Grad} \theta \geq 0.$$

All local relations following from (5)–(4) should be complemented by initial and boundary conditions. In particular, for c we assume the following boundary condition $\mathbf{n} \cdot \mathbf{j} = k(c - c^0)$, where k and c^0 are the diffusion parameter and concentration in an environment of the solid.

3. Reduction of 3D problem to 2D equations of shell thermodiffusion

In the shell-like body B , the boundary surface ∂B consists of three parts: the upper M^+ and the lower M^- shell faces as well as the lateral boundary surface ∂B^* . Then the position vector \mathbf{x} of B is described by $\mathbf{x} = \mathbf{x} + \zeta \mathbf{n}$, where \mathbf{x} is the position vector of some shell base surface M , \mathbf{n} is the unit normal vector orienting M , and $\zeta \in [-h^-, h^+]$. Each part $P(t)$ of the deformed shell-like body can be represented by a part $\Pi(t)$ of $M(t)$ defined by a position vector $\mathbf{y}(t)$. Applying the exact through-the-thickness integration [2, 6] of all 3D fields present in 3D laws of thermodiffusion given above, we can introduce uniquely the corresponding 2D fields now given entirely on M . Then following involved but straightforward transformations, all 3D laws of thermodiffusion can be reduced to their 2D representations, which can be regarded as exact implications of the 3D global laws. When 2D laws are supplemented by corresponding boundary and initial conditions, such resultant 2D initial-boundary-value problem for shell thermodiffusion becomes a direct extension of the resultant 2D model of shell thermodynamics developed in [2, 6]. As for the temperature, we introduce two surface diffusion-related fields which are related to the mean values of c and its gradient. As a result, instead of (5) we get two 2D equations for diffusion in shells.

As an example, we consider the hydrogen diffusion in pipelines in the special case of axisymmetric deformation and stress-assisted diffusion of a circular tube, [8].

References

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