ON BENDING OF A TWO-PHASE PLATE

V. A. Eremeyev[1] and V. Konopińska-Zmysłowska[1]

[1]*Gdańsk University of Technology, ul. Gabriela Narutowicza 11/12, 80-233 Gdańsk, Poland*

e-mail: eremeyev.victor@gmail.com; violetta.konopinska@pg.edu.pl

1. Introduction

Thin films made of shape memory alloys and polymers and other materials undergoing stress-induced phase transitions (PTs) constitute a new class of thin-walled structures, that is the two-phase plates and shells. For this class of shells the kinematics is complemented by additional state variables describing the phase change. Tubes, films, and plates made of martensitic materials are often used in MEMS as a basic working elements [3, 4]. Considering PTs in thin-walled structures and phase interfaces one may observe two cases of possible phase interfaces. The first case relates to the appearance and propagation of phase interfaces across the shell thickness, presented, e.g., in [8,9]. In this case the two-phase shell consists of two parts made of different phases separated by a phase interface curve. The second type of phase interfaces is that when the phase interface is almost parallel to the shell base surface, see, e.g., [11, 12]. Such two-phase shell treated as two-dimensional (2D) continuum within the so-called phase field approach as at any point both phases may exist.

For the first type of phase interfaces the phase equilibrium conditions in shells were established in [5] using variational approach within the six-parameter shell theory and were extended for thermo- and visco-elastic materials in [6, 7]. The background of the six-parameter shell model was developed in [10]. The aim of the lecture is to discuss the possible reduction techniques to the two-phase plates.

2. Equilibrium and quasistatic deformations 3D solids undergoing PTs

Following [1, 2] we briefly recall the compatibility conditions on a coherent phase interface in nonlinear elastic solids. In addition to the balances of stresses we have an additional thermodynamic condition which is required for the determination of a priori unknown phase interface $\mathcal{C}$. For statics the condition is given by

$$\llbracket \mathbf{n}\cdot\mathbf{B}\mathbf{n} \rrbracket = 0, \quad \mathbf{B} = W\mathbf{1} - \mathbf{P}\mathbf{F}^T, \tag{1}$$

where the double brackets denote the discontinuity jump of $\mathbf{n}\cdot\mathbf{B}\mathbf{n}$ across the phase interface, $\mathbf{B}$ is the Eshelby tensor, W the strain energy tensor, $\mathbf{P}$ the first Piola-Kirchhoff stress tensor, $\mathbf{F} = \nabla\mathbf{x}$ the deformation gradient, $\mathbf{x}$ is the position vector in an actual configuration, $\mathbf{n}$ the unit normal to $\mathcal{C}$, and $\mathbf{1}$ is the unit tensor. For quasistatic deformations instead of (1) we use the kinetic equation that is a relation between of the velocity V of the phase interface and the jump of $\mathbf{B}$ [2]

$$V = \Upsilon\left(\llbracket \mathbf{n}\cdot\mathbf{B}\mathbf{n} \rrbracket\right). \tag{2}$$

Here Υ is a kinetic function, $\Upsilon(0) = 0$. As a result, the boundary-value problem for solids undergoing PTs includes additional equations required for the determination of the position of the phase interface, that is the vector $\mathbf{x}_C$ which is independent on $\mathbf{x}$, in general.

3. Thermodynamic compatibility conditions for shells

The 2D analogues of (1) and (2) were established in [5–7], where the sharp interface model was assumed. In other words we consider a two-phase shell as a material surface consisting of two phases separated by smooth and a priori unknown phase interface $\mathcal{L}$. As a result, the thermodynamic compatibility condition for shells has the form (2) with new 2D Eshelby tensor $\mathbf{B}$ defined as

$$\mathbf{B} = U\mathbf{A} - \mathbf{N}^T\mathbf{F}_s^T - \mathbf{M}^T\mathbf{K}^T, \tag{3}$$

where U is the surface strain energy density, $\mathbf{N}$ and $\mathbf{M}$ are the surface stress measures of the 1st Piola-Kirchhoff type, $\mathbf{F}_s$ is the surface deformation gradient, $\mathbf{K}$ is the bending strain, and $\mathbf{A} = \mathbf{1} - \mathbf{n}\otimes\mathbf{n}$, see [5, 7] for details.

The 2D sharp interface model based on kinetic equation (2) with (3) can describe various peculiarities of PTs in thin-walled structures such as hysteresis loop [6, 7]. On the other hand for some kind of deformations of shell-like solids the new phase can appear as a layer, so instead of homogeneous shell we have layered structure, see, e.g., Fig. 1. Thus, one needs another 2D model of two-phase shells.

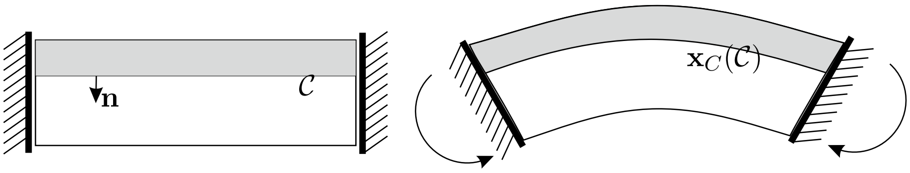

Figure 1: Deformation of a two-phase plate under bending: initial and bent configurations.

In order to describe the bending deformations of a plate undergoing PT, we utilize two approaches. The first approach is based on 3D analysis and the Eshelby tensor $(1)_2$. The position of $\mathcal{C}$ follows from the minimization of the energy functional. Then $\mathbf{N}$ and $\mathbf{M}$ have to be determined using the through-the-thickness integration. As a result, we get rather complex nonlinear problem. In particular, $\mathbf{N}$ and $\mathbf{M}$ became functionals of a priori unknown phase interface position. The second approach is based on the layer-wise model of the plate. We treat a plate undergoing bending as set of thin layers for each layer we introduce the 2D Eshelby tensor $\mathbf{B}_z$ such as given by (3). The criterion of PT is the value of $[\![\mathbf{n}\cdot\mathbf{B}_z\mathbf{n}]\!]$ in zth layer. This approach can be also more useful for multilayered films with thin layers made of different materials undergoing PTs. Few examples will be discussed during SOLMECH2018.

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