APPROXIMATION OF THE PARALLEL ROBOT WORKING AREA USING THE METHOD OF NONUNIFORM COVERING

Posypkin M.[1], **Usov A.**[1], **Rybak L.**[2], **and Malyshev D.**[2]

[1] *Dorodnicyn Computing Centre, FRC CSC RAS, Moscow, Russian Federation*
[2]*Belgorod State Technological University named after V.G. Shukhov, Belgorod, Russian Federation*
e-mail: rl_bgtu@intbel.ru

An important task, solved in the design of robots, is to determine its working area, i.e. set of points, which can reach tool, driven by a robot. The size of the working area is the key characteristic of the robot. The working area itself serves as the basis for laying the trajectory of the working tool. This article discusses two approaches to defining the working area of a robot.This article discusses two approaches to defining the working area of a robot DexTar [1]. One approach is based on the direct use of the system of kinematic equations of the coordinates of the output links along the lengths and angles of rotation of the links. The second approach is to bring a system of equations for links to a quadratic equation and verifying that the discriminant of the quadratic equation must not be less than zero.

Consider a flat RRRRR parallel robot DexTar (fig. 1a, b). The case is considered when the engines are located above the working area plane and not have on its influence.

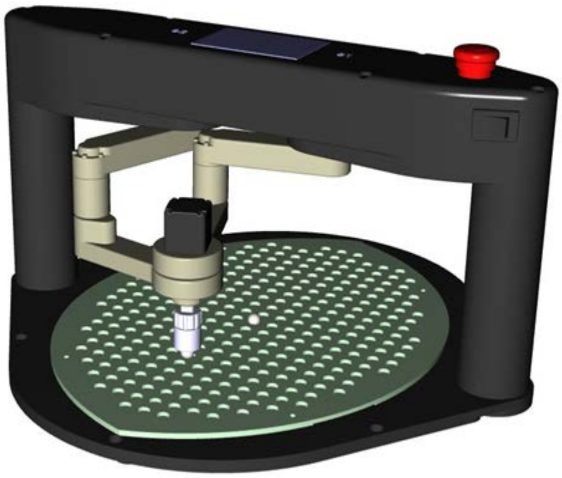

Fig. 1a. Robot RRRRR

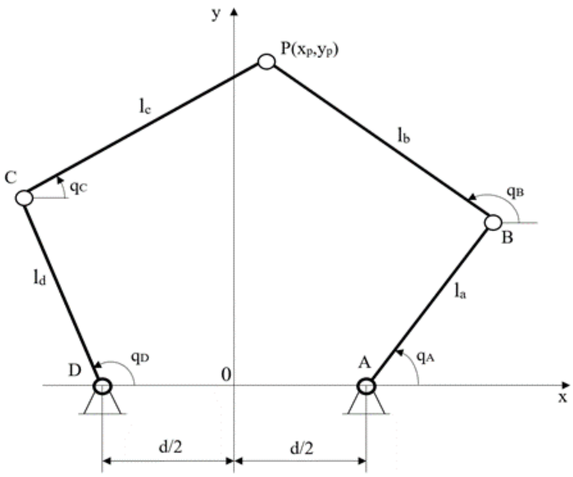

Fig. 1b. Diagram of the robot RRRRR

We write down the equations that determine the coordinate of the point $P(x_{p,}y_p)$:

$$\begin{cases} x_p = l_b \cdot \cos q_3 + l_a \cdot \cos q_1 + \frac{d}{2}, \\ x_p = -\frac{d}{2} + l_d \cdot \cos q_2 + l_c \cdot \cos q_4 , \\ y_p = l_a \cdot \sin q_1 + l_b \cdot \sin q_3 , \\ y_p = l_d \cdot \sin q_2 + l_c \cdot \sin q_4 . \end{cases} \quad (1)$$

These equations are the equations of connection of the robot under consideration and are formulas for computing the coordinates *x* and *y* according to the given geometric characteristics of the robot and angles of rotation of the links.

Consider two approaches for building a workspace using the method of uneven coatings. The first approach is based on the use of the system of equations (1). The algorithm works with two lists of six-dimensional parallelepipeds $\mathbb{P}$ and $\mathbb{P}_E$. Each of the axes of the six-dimensional space corresponds to changing parameters in the system of equations: coordinates x_p, y_p and angles of rotation of links q_i. The first step of the list $\mathbb{P}$ algorithm consists of only one parallelepiped P, which includes the entire range of angles $-\pi \le q_i \le \pi$ and ranges $-(l_a + l_b + d) \le x_p \le l_a + l_b + d$ and $-(l_a + l_b + d) \le y_p \le l_a + l_b + d$. Then the following actions are performed in the loop: A parallelepiped $P_i, i \in 1, n,$ is extracted from the list $\mathbb{P}$, where *n* is the number of parallelepipeds which determines the accuracy of the approximation. For him is consistently determined minimum and maximum of the equations functions $q_i, i \in 1,4$ from the following system, obtained from the solutions of the direct kinematics problem:

$$
\begin{cases}
x_p - l_b \cdot \cos q_3 - l_a \cdot \cos q_1 - \frac{d}{2} = 0, \\
x_p + \frac{d}{2} - l_d \cdot \cos q_2 - l_c \cdot \cos q_4 = 0, \\
\quad y_p - l_a \cdot \sin q_1 - l_b \cdot \sin q_3 = 0, \\
\quad y_p - l_d \cdot \sin q_2 - l_c \cdot \sin q_4 = 0.
\end{cases} \tag{2}
$$

If at least one of the equations q_i satisfies at least one of the conditions $(\min g_i > 0)$ or $(\max g_i < 0)$, consequently, the parallelepiped does not contain a combination of variables, which are solutions of the equation and are excluded from further consideration, falling into the list $\mathbb{P}_E$.

The second approach is based on the formula for calculating the discriminant of a quadratic function. In this approach, the algorithm works with three lists of two-dimensional parallelepipeds $\mathbb{P}, \mathbb{P}_I$ and $\mathbb{P}_E$. Each of the axes of a two-dimensional space corresponds to coordinates x_p and y_p. In the first step of the algorithm, the list P also consists of only one parallelepiped P, including ranges $-(l_a + l_b + d) \leq x_p \leq l_a + l_b + d$ and $-(l_a + l_b + d) \leq y_p \leq l_a + l_b + d$.

We write the conditions, taking into account that $x'_{P1} = x_p - d/2$ and $x'_{P2} = x_p + d/2$:

$$
\begin{cases}
(2\left(-\frac{x_p - d/2}{y_p}\right)\left(\frac{(x_p-d/2)^2}{2y_p} + \frac{y_p}{2} + \frac{A^2-B^2}{2y_p}\right))^2 - 4(1 + \frac{(x_p-d/2)^2}{y_p^2})((\frac{(x_p-d/2)^2}{2y_p} \\
\qquad + \frac{y_p}{2} + \frac{A^2-B^2}{2y_p})^2 - A^2) >= 0 \\
\left(2\left(-\frac{x_p+d/2}{y_p}\right)\left(\frac{(x_p+d/2)^2}{2y_p} + \frac{y_p}{2} + \frac{A^2-B^2}{2y_p}\right)\right)^2 - 4(1 + \frac{x_p+d/2)^2}{y_p^2} \\
\qquad + \frac{y_p}{2} + \frac{A^2-B^2}{2y_p})^2 - A^2) >= 0
\end{cases} \tag{3}
$$

From the list $\mathbb{P}$ has extracted a parallelepiped $P_i, i \in 1, n$. For him to consistently define the minimum and maximum of the function g_i of the systems of inequality (3). If for both functions condition $(\min g_i \geq 0)$, then it completely satisfies the conditions and is added to the list $\mathbb{P}_I$. If at least one function meets a condition $(\max g_i < 0)$, then consequently, parallelepiped does not meet the requirements and is excluded from consideration, falling into the list $\mathbb{P}_E$. In other cases, the parallelepiped is divided into two equal parallelepipeds on a method similar to the first approach - along the rib with the longest length. These parallelepipeds are entered at the end of the list $\mathbb{P}$. The loop is executed *n* times which is specified when the program is started. Determination of the minimum and maximum of functions in both cases was carried out by the method of uniform search by grid search and interval analysis method. The algorithm is implemented in C ++ using the Snowgoose library. The simulation revealed inaccuracies in determining the maximum and minimum for the second approach by the interval analysis method as a result of the occurrence of one variable in the calculations several times. Compensate for this error to ensure the accuracy of the work area ± 0*;* 2 can first divide the work area into 16,777,216 parallelepipeds. In addition, using the method of uniform search for the first approach, it takes a lot of time to compute, considerably exceeding the time of calculation by the method of interval analysis, with a high degree of evaluation of more than 10 times. Based on the foregoing, it is found that the most effective use of the method interval analysis for the first approach, based on the system of equations (1), uniform grid search method for the second approach, based on the system of equations (3).

This work was supported by the Russian Science Foundation, the agreement number 16-19-00148.

[1] Sergiu-Dan Stan Vistrian Maties, Balan Radu. *Optimization of a 2 dof micro parallel robot using genetic algorithms*. Frontiers in Evolutionary Robotics. 2008. P. 465–490.