# WRINKLING PATTERN TRANSITION OF AXIALLY COMPRESSED BILAYERED CYLINDRICAL COMPOSITES

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## **Problem description**

Surface wrinkling is observed in diverse composite systems, ranging from biology to engineering. Wrinkling usually develops due to the loss of system's stability. Traditionally, it has been understood as a sign of failure. In recent years, however, it is exploited to achieve advanced functionality or superior physical properties of a composite system, e.g. [1]. A systematic understanding of the process of development and evolution of the wrinkling pattern calls for extensive numerical analyses. In this respect, it turns out that accurate numerical solutions related to wrinkling simulations are difficult to attain. Specialized numerical formulations have been developed.

In this work, we study wrinkling of the stiff shell-soft core cylindrical systems. The elastic cylinder is resting on elastic substrate with no inside cavity. It is exposed to axial compression as shown in Fig. 1.

Experimental, theoretical and numerical investigation of a system on Fig. 1 was reported in [2] and [3]. The authors showed that regardless of the system characteristics, the first wrinkling mode is always axisymmetric. They also observed wrinkling pattern transition from the axisymmetric to the diamond-like pattern for some configurations. They proposed a coefficient  $C_S$  with the critical value  $C_{S,crit} \approx 0.88$ , see Eq. (1), where  $E_F$  and  $E_S$  are elastic moduli of film and substrate, respectively. They showed that for  $C_S < C_{S,crit}$ , only the axisymmetric wrinkling mode occurs, whereas for  $C_S >$  $C_{S.crit}$ , the transition from the axisymmetric to the diamond-like wrinkling mode is expected.



Figure 1: Problem description

(1) 
$$C_S = \frac{E_S}{E_F} \left(\frac{R}{t_c}\right)^{\frac{3}{2}}$$

### **Solution methods**

In [2] and [3], numerical solutions of the Fig.1 problem were obtained by rather complex 3d solid and composed 3d solid and shell models. We recomputed the problem by a simpler and faster numerical formulation. The shell was modelled by 6-parameter stress-resultant extensible-director shell quadrilateral finite element, enhanced by ANS (Assumed Natural Strain) concept. The substrate was modelled as a set of linear springs, with stiffness  $K_s$ . As a solution method, implicit dynamics was chosen. We used three implicit time-stepping schemes with controllable numerical damping of high-frequency modes and little numerical dissipation in the low-frequency range: generalized  $\alpha$  scheme [4], JHW scheme [5], and Energy-Momentum-Decaying scheme [6].

#### Results

In Fig. 2 some results are shown that were computed with generalized  $\alpha$  method (GAM). The transition from axisymmetric wrinkling pattern to the diamond-like pattern was nicely captured.



Figure 2: Two sets of input data (left), response graphs (middle) and wrinkling patterns (right)

#### References

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