LOVE WAVES PROPAGATION IN ELASTIC WAVEGUIDES LOADED BY VISCOELASTIC MEDIA

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1. Introduction

It is very important from a practical point of view, to develop new and accurate methods of measuring the rheological parameters (viscosity η, elasticity μ and density ρ) of plastics and polymers. New materials require new methods of measuring their rheological parameters. To evaluate the rheological parameters of plastics so far mechanical methods are used. These methods are cumbersome, outdated and destructive. The use of SH (Shear Horizontal) surface Love waves, to evaluate rheological parameters of polymers, does not possess these disadvantages. The objective of this work is to establish a mathematical model of propagation of Love waves in layered elastic waveguides covered on their surface with viscoelastic materials described by different viscoelastic models, i.e., Kelvin-Voigt, Newton and Maxwell models. To this end, we developed a complex dispersion equation for Love waves propagating in loaded waveguides and performed numerical calculations.

2. Physical model

Dispersion curves of the Love wave, i.e., the dependence of the phase velocity and attenuation on frequency, result from solution of the boundary value problem (called the direct Sturm-Liouville problem). Love waves propagate in layered structures, e.g., in elastic waveguides composed of an elastic surface layer rigidly bonded to an elastic substrate. The top of the surface layer $x_2 = -D$ is loaded with a viscoelastic medium, see Fig.1. Love waves, propagating in isotropic waveguides, have only one SH (Shear Horizontal) component of vibrations u_3, polarized along the axis x_3 that is perpendicular to the direction of propagation x_1.

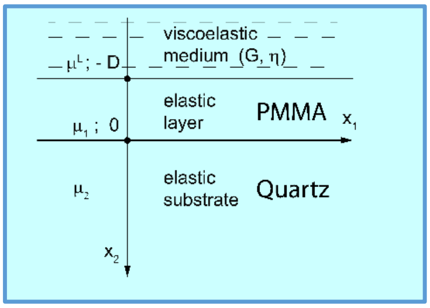

Fig.1. Lossless (elastic) Love wave waveguide (surface layer plus substrate) loaded at the surface $x_2 = -D$ with a lossy viscoelastic medium of the shear modulus G and viscosity η.

3. Rheological models of viscoelastic media

We chose 3 models for the viscoelastic media, i.e., Newton, Kelvin-Voigt, and Maxwell models, that load the surface of the Love wave waveguide. For time-harmonic waves, using constitutive equations for the 3 visco-elastic media considered, we obtain the following 3 formulas for the complex shear moduli c_{44}^L of elasticity:

a) Newton model: (1) $c_{44}^L = -j\omega\eta$; where: η is the viscosity of the viscoelastic medium,

b) Kelvin-Voigt model: (2) $c_{44}^L = G - j\omega\eta = G(1 - j tan\delta)$; where: G is the elastic shear modulus, and η is the viscosity of a viscoelastic medium, $tan\delta = \omega\eta/G$, and

c) Maxwell model: (3) $c_{44}^L = G\frac{(\omega\tau)^2}{1+(\omega\tau)^2} - jG\frac{\omega\tau}{1+(\omega\tau)^2}$; where: $\tau = \eta/G$ is the relaxation time.

4. Complex Dispersion Equation for Love Waves

The complex dispersion equation for Love waves that propagate in the waveguide structure from Fig.1 is:

$$\sin(qD) \cdot \{(\mu_1)^2 \cdot q^2 - \mu_2 \cdot b \cdot \lambda_1 \cdot c_{44}^L\} - \cos(qD) \cdot \mu_1 \cdot q \cdot \{\mu_2 \cdot b + \lambda_1 \cdot c_{44}^L\} = 0 \tag{4}$$

where: complex pairs of quantities $(\mu_{2,}\ b)$, $(\mu_{1,}\ q), and\ (\lambda_1, c_{44}^L)$ correspond to the substrate, surface layer and the loading material, respectively, and $j = (-1)^{1/2}$. The above equation is a nonlinear transcendental algebraic equation for the complex propagation constant $k = k_0 + j\alpha$ as unknown. Solving equation 4 (the solution is a pair $(k_0,\ \alpha)$) allows for determination of the phase velocity of the Love wave $v = \omega/k_0$ and the imaginary part α of the complex wavenumber k that represents the attenuation in Np/m of the Love wave per unit length.

5. Dispersion Curves of Love Waves

Numerical calculations were performed for the waveguide structure shown in Fig.1. The substrate is Quartz (ST-cut 90º X) and the surface layer is PMMA poly(methyl methacrylate). The frequency of the Love wave varied from 1 to 1000 MHz. Thickness D of the surface layer equaled 0.1 mm. Losses in the PMMA layer and Quartz substrate were neglected. The only source of losses is the viscosity of the viscoelastic medium.

Attenuation Curves

In Figure 2 the attenuation of Love waves, in the range of frequencies; from 0 to 1000 MHz is presented.

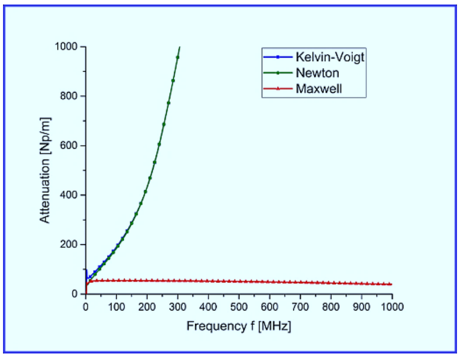

Fig.2. Attenuation of the Love surface wave, propagating in a lossless elastic waveguide, loaded with 3 different types of lossy viscoelastic materials, i.e., Kelvin-Voigt, Newton and Maxwell. Low, medium and high frequency limits: $tan\delta \in [0.127 - 127]$, $G = 5 \cdot 10^4 Pa,\ \eta = 1\ mPas$.

6. Conclusions

The theoretical analysis and the results of numerical calculations presented in this paper reveal that the attenuation of the Love wave reflects directly the viscoelastic properties of the loading material described by Kelvin-Voigt, Newton and Maxwell models. Namely:

a) in the low frequency limit $tan\delta \ll 1$ the attenuation of the Love wave due to the Maxwellian liquid and that due to the Newtonian liquid are almost the same (see Fig.2)

b) in the high frequency limit $tan\delta \gg 1$ the attenuation of the Love wave due to the Kelvin-Voigt material and that due to the Newtonian liquid are almost identical (see Fig.2).

The results of this study should be useful for designers and scientists working in geophysics, microelectronics (MEMS, biosensors, chemosensors), mechanics of materials and biomechanics.

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