AN APDM+PTM APPROACH FOR EVALUATING THE RESPONSE PDF OF UNCERTAIN STRUCTURAL SYSTEMS

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It is known that there is no universal method suitable to solve any problem involving uncertainties in structural system characteristics. One of the oldest method for the evaluation of the response pdf of systems subjected to uncertainties is based on truncating the cumulant series expansion of the response characteristic function [1], which is the Fourier transform of the pdf. This method provides good results if the response is characterized by a relatively low non-Gaussianity. When the response is strongly non-Gaussian, the number of terms of the series may be particularly high and the convergence, that is not guaranteed, can be particularly slow. In addition, the direct evaluation of the terms of the series may not be simple.

Widely used methods are those based on the perturbation approaches, based on a Taylor series expansion in terms of a set of zero mean random variables. The perturbation approaches provide accurate results for relatively low levels of uncertainty, for which only few terms of the series are used (usually the first or the first and second order are considered). On the contrary, when the level of uncertainty of the structural parameters increases the approach loses strongly its precision and, moreover, the computational effort increases exponentially because a high number of terms of the series must be taken into account. In any case, the convergence of the approach is not guaranteed by the augmented order of the retained series terms [2].

Another important class of methods for solving uncertain structural systems is that based on the projection approaches, that is on the projection of the solution on a complete stochastic basis. Two of the most used projection approaches are those based on the Karhunen-Loève expansion and on the polynomial chaos expansion [3]. This last one is a Galerkin projection scheme based on Wiener integral representation. It requires the numerical evaluation of the series expansion terms and can be particularly onerous if the terms of the series are not limited to a relatively small number. For this reason recently several efforts has been made to improve the approach [4].

A relevant class of methods dealing with uncertain system is that related to the use of the random matrix expansion of the structural stiffness matrix in order to perform explicitly its inversion (Neumann expansion) [5]. Then, once that the explicit inverse stiffness matrix is known, it is possible to obtain the statistics of the response, or to perform a MCS to obtain the response pdf.

In 2002 Falsone and Impollonia [6,7] proposed the APDM, that belongs to the class of MCS-based methods [8]. It consists in breaking up the structural response in the base of the main deformation modes of the structure: this allows obtaining an approximation of the response, without the cost to invert the stiffness matrix of the system and enabling to reduce strongly the computational effort, the statistics of the response being obtained by the MCS directly applied on the explicit expressions of the response. In a certain sense, the method enables the evaluation of an approximated inverse stiffness matrix (as the matrix expansion methods). Nevertheless, the APDM can be considered also as a projection method, because it consists essentially in the expansion of the structural response on a particular base through a finite number of functions, depending on the uncertain parameters, strictly related to the principal deformation modes of the structural system. In any case, the coefficient of the series can be evaluated explicitly in terms of the uncertain parameters. This method is remarkably efficient, allowing the probabilistic analysis of systems with higher levels of uncertainty than that related to an efficient use of the perturbation methods.

Recently, an approach, based on a new version of the PTM, has been proposed for the study of some stochastic problems [9,10]. The method provides the basis for a new philosophy in the study of structural systems subjected to random loads, working directly in terms of input and output pdfs.

Aim of the present paper is matching the APDM with the PTM in order to give an approach able to characterize the response of uncertain structural systems directly in terms of pdf and without using any expansive MCS. Here, a new APDM+PTM approach is proposed in order to perform the stochastic response analysis of FE modelled structural systems affected by uncertainties which are represented by random variables having any kind of joint pdf. In this way, the attempt of taking into account the virtues of both APDM and PTM is made. The result is in the definition of a very efficient approach from both the accuracy and the computational effort point of view. Moreover, it is shown that, for statically determinate structure the APDM becomes PDM, in the sense that it is able to give the exact relationships between the uncertain parameters and the structural response. As the application of the PTM can be made without introducing no more approximation, then the PDM+PTM approach is able to give the exact response pdfs of statically determinate uncertain structural systems.

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