BUCKLING OF THIN-WALLED STRUCTURES WITH SHAPE MEMORY EFFECT UNDER THERMOELASTIC PHASE TRANSITIONS

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1. Introduction

It was shown in [1] that NiTi columns cooled homogeneously down to the austenite-to-martensite phase transition buckles under the compression dead load of 10%-15% of the Eulerian force corresponding to the minimum modulus even though the pure martensite is not reached. Thus, the traditional buckling analysis fails, and the dropping of elastic moduli cannot be interpreted as a main buckling cause for SMAs, moreover the same phenomenon is observed during the martensite-to-austenite transition following by raising moduli. Consistent critical load estimates for systems with phase transitions must be based only on coupled models [3]. The corresponding buckling concepts of von Kàrmàn's and Shenley's types were proposed in [4]; the good correlation with the test data was obtained. Here the free of concepts 3D numerical solutions for buckling and postbuckling of thin-walled SMA elements are presented, and their correlation with various analytical estimates [4] is shown.

2. Coupled model of thermoelastic SMA behavior

First, the dependence of the martensite volume ratio q on the dimensionless temperature t is given by (1) [2,5]:

$$\begin{gathered} q=\tfrac{1}{2}\left(1-\cos\pi t\right), \quad t\in[0,1]; \quad t\leqslant 0 \;\Rightarrow\; q=0; \quad t\geqslant 1 \;\Rightarrow\; q=1; \\ A\to M:\; t=(M_s-M_f)^{-1}\left(M_s-T_\sigma\right); \quad M\to A: \quad t=1-(A_s-A_f)^{-1}\left(A_s-T_\sigma\right). \end{gathered} \tag{1}$$

M_s, M_f are the start and finish temperatures of the $A\to M$ transition and A_s, A_f are the corresponding temperatures for $M\to A$ in unstressed state. The stress effect is accounted by the reduced temperature T_σ (2):

$$T_\sigma=T-\Delta S^{-1}\left[\omega_{ij}s^{ij}+Z(\sigma^{ij})+\varepsilon_0\varepsilon_k^k\right], \quad Z(\sigma^{ij})=\tfrac{1}{2}(\sigma_k^k)^2\left(K_M^{-1}-K_A^{-1}\right)+\tfrac{1}{6}\sigma_i^2\left(G_M^{-1}-G_A^{-1}\right); \tag{2}$$

$s^{ij}=\sigma^{ij}-\frac{1}{3}g^{ij}\sigma_k^k$, ΔS is the volumetric entropy density drop between A and M at a reference temperature, ε_0 is the $A\to M$ volumetric effect, and G_M, G_A, K_M, K_A are shear and bulk moduli of pure M and A phases,

$$A\to M: \quad \omega_{ij}=(2+q)^{-1}\left[3\rho_D s_{ij}\sigma_i^{-1}\varphi_1(\sigma_i)+e_{ij}^{(1)}\right]; \qquad M\to A: \quad \omega_{ij}=q^{-1}e_{ij}^{(1)}. \tag{3}$$

The strain superposition is assumed, thus, $\varepsilon_{ij}=\varepsilon_{ij}^{(0)}+\varepsilon_{ij}^{(1)}$, where $\varepsilon_{ij}^{(0)}$ is the elastic strain given by (4) [2,5]:

$$\begin{gathered} \varepsilon_{ij}^{(0)}=\tfrac{1}{2}G^{-1}\sigma_{ij}-\left(\tfrac{1}{6}G^{-1}-\tfrac{1}{9}K^{-1}\right)\varepsilon_k^k g_{ij}+\alpha T g_{ij}; \\ G^{-1}=qG_M^{-1}+(1-q)G_A^{-1}; \quad K^{-1}=qK_M^{-1}+(1-q)K_A^{-1}; \quad \alpha=q\alpha_M+(1-q)\,\alpha_A, \end{gathered} \tag{4}$$

The increment of the phase and structure strain deviator $e_{ij}^{(1)}=\varepsilon_{ij}^{(1)}-\frac{1}{3}g_{ij}g^{kl}\varepsilon_{kl}^{(1)}$ can be defined as follows (5):

$$A\to M: \quad de_{ij}^{(1)}=\omega_{ij}dq+\tfrac{3}{2}\rho_D s_{ij}\sigma_i^{-1}q\psi_2(\sigma_i)d\sigma_i; \qquad M\to A: \quad de_{ij}^{(1)}=\omega_{ij}dq; \tag{5}$$

ρ_D is the maximum strain intensity of the $A\to M$ [2]. The increments $dq(d\sigma^{kl},dT)$, $d\varepsilon_{ij}(d\sigma^{kl},dT)$ can be formulated hence, therefore we obtain $R_{ijkl}=\partial\varepsilon_{ij}/\partial\sigma^{kl}$ and $\alpha_{ij}=\partial\varepsilon_{ij}/\partial T$ allowing the tangent stiffness C^{ijkl} computing at a step of any algorithm, i. e. the model (1-5) can be implemented into finite element codes.

3. Buckling of SMA elements undergoing thermally induced martensite transitions

Let us consider a SMA solid with an initial configuration $G \in \mathbb{R}^3$, $\partial G = S_\sigma \oplus S_r$ defined by a vector radius $\mathbf{r}(M)$ with kinematical constraints $\mathbf{r}(M \in S_r) = \mathbf{r}_\star$ and loads $\mathbf{p}(M \in S_\sigma)$, and let it be entirely austenite: $q = 0$, $T_\sigma \geqslant M_s$. The further cooling is assumed to be sufficiently slow to neglect the heat conduction. Let us apply the perturbation $\delta\mathbf{r}$, $\delta\mathbf{r}(M \in S_r) = 0$; the stable and unstable configurations can be defined by (6), (7):

$$\forall \varepsilon > 0 \quad \exists \delta(\varepsilon) > 0 : \quad \forall \|\delta\mathbf{r}\| < \delta \quad \|\mathbf{u}(\delta\mathbf{r}, \tau, \mathbf{p})\| < \varepsilon; \tag{6}$$

$$\exists \tau^\star(\mathbf{p}) : \quad \forall \tau > \tau^\star \quad \|\mathbf{u}(\delta\mathbf{r}, \tau, \mathbf{p})\| \geqslant \varepsilon^\star, \tag{7}$$

where $\varepsilon^\star$ is the representative displacement value and the bifurcation point $\tau^\star$ can be found by analyzing the equilibrium curve or as the kinetic energy maximum point corresponding to the quick equilibrium state change:

$$\mathrm{T}(\tau^\star) = \tfrac{1}{2}\rho \dot{u}^i(\tau^\star)\dot{u}_i(\tau^\star) = \mathrm{T}_{\max}, \quad T_\star = T(\tau^\star). \tag{8}$$

here $T_\star$ is the critical temperature. This approach corresponds to the assumed "supplementary phase transform occurring everywhere", i. e. Shenley's concept. On the other hand, instantaneous applying of $\delta\mathbf{r}$ under fixed temperature $T \leqslant T_0$ and fixed load $\mathbf{p}$ should be close to the "fixed load assumption", or von Kàrmàn's concept.

4. Results and conclusions

Several finite element simulations for NiTi beams and plates were performed on the bsis of the SMA model (1–5) and the buckling defined by (6–8). The buckling forces obtained for the clamped-clamped prismatic NiTi beam of 0.01 m length and with 0.002×0.001 m cross-section are compared with the traditional Euler estimate P_{Cr} corresponding to the minimum elastic modulus of the martensite $E_{\mathrm{M}} = 30$ GPa for thermally induced and superelastic $A \to M$ transforms (table 1). The phase transition regime depends on the initial temperature T_0.

$P_{\mathrm{Cr}}(E_{\mathrm{M}})$, N	Thermally induced:	$P(\tau^\star)$, N	T_0, K	Superelasticity:	$P(\tau^\star)$, N	T_0, K
1974		220	343.15		375	323.15

Table 1: Buckling forces for the clamped-clamped prismatic NiTi beam of 0.01 m length: Euler's estimate based on minimum elastic modulus, E_{M}; thermally induced $A \to M$ transition; superelastic $A \to M$ transition.

The computed loads $P(\tau^\star)$ are about 10…15% of Euler's ones $P_{\mathrm{Cr}}(E_{\mathrm{M}})$, are consistent with the test data [1] and analytical solutions corresponding to the extended Shenley concept [4], and the buckling forces for the thermally induced $A \to M$ transform are about only 60% of the critical forces for the superelasticity regime. Thus, the buckling danger for SMA elements undergoing thermally induced phase transitions is underestimated, and the traditional bifurcation analysis fails whereas the approach [3, 4] allows one to obtain adequate results.

Acknowledgments This investigation is the part of the State Task for Basic Researches GP 14 (0050-2017-0001), the state register number AAAA-A17-117032010136-3, and partially supported by the RFBR under grant No. 16-01-00751-a.

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