DETERMINATION OF VIBRATIONS IN AN ELASTIC MEDIUM AFTER THE PASSAGE OF A SPHERICAL WAVE THROUGH A VIBRATION-ABSORBING PLATE

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Introduction

In current times buildings and facilities are exposed to various external negative effects arising from utility equipment, industrial machinery, as well as transportation means (such as shallow-depth city railroad systems, heavy trucks, railway trains, trams) that cause huge dynamic loads [1]. Vibration-absorbing barriers placed between the vibration source and the object to be protected is one of the ways of protection of foundations against ground vibrations [2, 3].

This paper is intended to study vibration-absorbing properties of a plate exposed to subsurface spherical harmonic waves. In practice, such situation may correspond to vibrations caused by a point source situated nearby the barrier.

1. Statement of problem

Let us consider an elastic plate surrounded from both sides by half-spaces “1” and “2” filled with the ground. The coordinate system $Oxyz$ is of Cartesian type. It is assumed that the plane Oxy for the plate is a median one and limited along axes Ox and Oy ,having the size l x l . The beginning of the coordinates is assumed to be situated in the upper right corner of the plate.

When undisturbed, the ground is considered to be undeformed. The obstacle is overrun by a harmonic tensile wave of an normal stress amplitude p_* at the front and frequency ω , coming from the negative Oz - axis. The normal vector towards the front of the wave is in the plane Oxy .

The main purpose of this study is finding the resultant vector field of acceleration **a** (vibration accelerations) in the second half-space as a function of the frequency ω and spatial coordinates x, y and z depending on the parameters of the plate. The stated purpose of the problem solution is refined as follows. It is necessary to find the coordinates of the vibration acceleration field

(1) $$a_x = -\omega^2 u_1^{(2)},\ a_y = -\omega^2 u_2^{(2)}, a_z = -\omega^2 w^{(2)},$$

and the module

(2) $$a = \sqrt{a_x^2 + a_y^2 + a_z^2}\ ,$$

where $u_1^{(2)}, u_2^{(2)}$ are displacements of the medium “2” by coordinates x, y ; $w^{(2)}$ is the normal displacement of the second medium.

The mathematical statement of the problem includes setting the ingoing wave, equations of displacements of the ground and plate, boundary conditions for the plate and ground, condition for infinity, as well as a condition of the ground-to-obstacle contact where the adhesion of the plate to the ground is ignored.

2. Equations of displacements of sandwich plate

The displacement of the plate is described by a system of Paymushin V.N. equations [4]. Two bearing layers are of elastic isotropic type with the modulus of elasticity E , Poisson’s ratio ν , and thickness of $2t$. The filler is orthotropic of a honeycomb configuration with the elasticity modulus E_z , the Poisson’s ratio ν_z , and thickness of $2h$. The bearing layers are affected by normal outer stress loads p_1 and p_2 . Tangential displacements are indicated by $u_1^{(k)}$ and $u_2^{(k)}$ along the axes Ox and Oy. $w^{(k)}$ is the normal displacement of the bearing layer, q^1 and q^2 are transverse tangent lines of the stress in the filler by axes *Ox* and *Oy*. The Paymushin V.N. equation system which describes the movement of the plate has the following form:

$$
\begin{gathered}
\rho_c \ddot{u}_1^c = L_{11}\left(u_1^c\right) + L_{12}\left(u_2^c\right),\ \rho_c \ddot{u}_2^c = L_{21}\left(u_1^c\right) + L_{22}\left(u_2^c\right), \\
\rho_a \ddot{u}_1^a = L_{11}\left(u_1^a\right) + L_{12}\left(u_2^a\right) + 2q^1,\ \rho_a \ddot{u}_2^a = L_{21}\left(u_1^a\right) + L_{22}\left(u_2^a\right) + 2q^2, \\
\rho_c \ddot{w}_c - \underline{m_c \Delta \ddot{w}_c} + \underline{\rho_{wq}\left(\ddot{q}^1_{,x} + \ddot{q}^1_{,y}\right)} = -D\Delta^2 w_c + 2k_1\left(q^1_{,x} + q^1_{,y}\right) + p_1 - p_2, \\
\rho_{aw} \ddot{w}_a - \underline{m_a \Delta \ddot{w}_a} = -D\Delta^2 w_a - 2c_3 w_a + p_1 + p_2, \\
\underline{\rho_{q1}\ddot{q}^1} - \underline{\rho_{wq1}\ddot{w}_{c,x}} = u_1^a - k_1 w_{c,x} - k_2\left(q^1_{,x} + q^2_{,y}\right)_{,x} + k_{31} q^1, \\
\underline{\rho_{q2}\ddot{q}^2} - \underline{\rho_{wq2}\ddot{w}_{c,y}} = u_2^a - k_1 w_{c,y} - k_2\left(q^1_{,x} + q^2_{,y}\right)_{,y} + k_{32} q^2;
\end{gathered} \tag{3}
$$

where $u_i^c = u_i^{(1)} + u_i^{(2)}, u_i^a = u_i^{(1)} - u_i^{(2)}\ (i = 1,2), w_c = w^{(1)} + w^{(2)}, w_a = w^{(1)} - w.$

The boundary conditions correspond to a hinge edge of the plate. All functions change harmonically.

The plate's kinematic parameters are represented in the form of two-fold trigonometric series meeting the boundary conditions. The amplitudes of ingoing and passed waves are expanded into series in a similar way. The solution of an equation system consists in finding the values of the kinematic parameters depending on amplitudes of wave pressure in mediums "1" and "2".

2. Equations of ground displacements and ingoing wave

A homogeneous elastic isotropic medium is used as a model of the ground [5], which equals the displacement equations (Lame equations) and the equations with respect to scalar potential φ and vector potential $\boldsymbol{\psi} = \left(\psi_1, \psi_2, \psi_3\right)$ of elastic displacements. As the area occupied by the ground is boundless, the potentials of equation solutions must satisfy the Sommerfeld radiation conditions:

To set an ingoing spherical harmonic wave, a spherical wave is considered [5] which travels along the positive direction of Oz axis.

The values of the potentials for the ingoing spherical harmonic wave are substituted into the equations of motion of the elastic medium. Taking into account that $\sigma_{33}\big|_{t=0,\ z=0} = p_*$, we will obtain the formula of amplitudes of displacements, deformations and stresses in the ingoing wave.

3. Boundary problem of interaction of harmonic wave with plate in ground

The boundary conditions with respect to the coefficients of the series depend on the conditions of contact between the plate and the ground To find the coefficients of the series corresponding to a disturbed stress-strained condition in the media, let us plug the potentials expanded into two-fold trigonometrical series. Introducing the constant values obtained from the contact conditions into the displacement expressions, we obtain the coefficients of the expansion in the series of displacements in the medium "2"Then it became possible to find the vibration acceleration module and its components.

References

[1] Umek A. Dynamic responses of building foundations to incident elastic waves.-Phd Thesis Illinois Institute of Technology, Dec.1973

[2] Kostrov B.V. Movement of rigid solid plate sealed into elastic medium affected by flat wave. PMM, 1964, V.28, issue.1 [in Russian]

[3] Rylko M.Y. [On movement of rigid rectangular inclusions in elastic medium affected by flat wave]. MTT,1977, No I [in Russian]

[4] Ivanov V.A., Paymushin V.N. Refined statement of dynamic problems of sandwich shells with transversal soft fillings and numerical analytical method of their solution. Prikladnaya mekhanika i tekhnicheskaya phisika [Applied Mechanics and Physics], 1995, V.36, No4, p. 147-151 [in Russian]

[5] Gorshkov A.G., Medvedsky A.L., Rabinsky L.N., Tarlakovsky D.V. Waves in continuum media: Textbook for Universities. Moscow, FIAMATLIT, 2004, 472 p. [in Russian]