THERMAL STRESSES AND TWO TEMPERATURE HEAT TRANSFER

Włodzimierz Bielski[1], Piotr Kowalczyk[2], and Ryszard Wojnar[3]

[1]*Institute of Geophysics PAS, Warszawa, Poland*
[2]*Institute of Applied Mathematics and Mechanics, University of Warsaw, Warszawa, Poland*
[3]*Institute of Fundamental Technological Research PAS*

e-mail: wbielski@igf.edu.pl

1. Introduction

For description of thermal behaviour of metal films exposed to ultrashort laser pulses a two-temperature heat theory was introduced, cf. [1] - [4]. The theory involves the electron temperature $T_{\rm e}$ and the lattice temperature $T_{\rm a}$. The energy transferred by electrons to lattice per unit volume of the crystal per unit time is proportional to the difference $T_{\rm e} - T_{\rm a}$. Mathematical properties of the theory were discussed in [5] and [6]. The problem has analogies in the diffusion theory (two-level diffusion of enzymes) and in viscous flow through non-homogeneous porous media with components of different porosity, [7].

2. Basic heat equations

Consider an elastic solid body B (a crystal, in particular a metal) irradiated by a laser pulse. Emerging of equilibrium between electrons and lattice in a crystal is realized by a relaxation process. During the relaxation processes crystal must be looked upon as a two-temperature system. The heat energy transferred by electrons to lattice per unit volume of the crystal per unit time is

$$\alpha(T_{\rm e} - T_{\rm a}) \quad \text{with} \quad \alpha \propto \frac{s^2}{\tau_{\rm e}\, T_{\rm e}} \tag{1}$$

where $T_{\rm e} = T_{\rm e}(x, t)$ is the electron temperature and $T_{\rm a} = T_{\rm a}(x, t)$ - the lattice temperature, both being functions of the position $\boldsymbol{x}$ and time t. Moreover, s denotes the velocity of sound, and $\tau_{\rm e}$ is the electron relaxation time, which can be regarded as a function of the electron temperature $T_{\rm e}$. The expressions (1) are valid, when the temperatures $T_{\rm e}$ and $T_{\rm a}$ are much greater than Debye's temperature. Since, in this case $\tau_{\rm e} \propto 1/T_{\rm e}$, the heat transfer coefficient α is independent of the temperature.

Estimates based on the electrical conductivity of metals give values of α of the order 10^{10} J/(cm^3 s K). If the specific heat of the lattice is $c_{\rm a}$, the relaxation time for the phonon temperature is of the order of $c_a/\alpha \approx 10^{-10}$s. For laser pulses of shorter duration, the violation of equilibrium between the electrons and lattice becomes important.

The heat energy balance equations for the metal absorbing the laser pulse has the form

$$\begin{aligned} c_{\rm e}(T_{\rm e})\frac{\partial T_{\rm e}}{\partial t} &= \lambda_{\rm e}\,\Delta T_{\rm e} - \alpha\,(T_{\rm e} - T_{\rm a}) + r(\boldsymbol{x}, t) \\ c_{\rm a}(T_{\rm a})\frac{\partial T_{\rm a}}{\partial t} &= \lambda_{\rm a}\,\Delta T_{\rm a} + \alpha\,(T_{\rm e} - T_{\rm a}) \end{aligned} \tag{2}$$

where λ denotes the heat conductivity coefficient, and the function $r(\boldsymbol{x}, t)$ describes the energy brought by the laser pulse. The symbol Δ denotes Laplacian.

3. Thermal deformations and stresses

We shall regard as the non-deformed state the state of the body before applying the heat source and consider deformations which accompany changes in the temperature of the body $T_{\rm a} - T_0$. The stress tensor is, cf. [8].

$$\sigma_{ij} = 2\mu\, u_{ij} + \left(K - \frac{2}{3}\mu\right) u_{ll}\, \delta_{ij} - \gamma(T_{\rm a} - T_0)\delta_{ij} \quad \text{with} \quad u_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{3}$$

and $(u_i) = \boldsymbol{u}$ is the displacement in the elastic body. Moreover, K is the bulk modulus (modulus of hydrostatic compression) and μ is the shear modulus (modulus of rigidity). The coefficient $\gamma = 3K\alpha_{\rm T}$, and $\alpha_{\rm T}$ is the thermal expansion coefficient of the body. The equation of motion reads

$$\rho\frac{\partial^2 \boldsymbol{u}}{\partial t^2} = 2\mu\, \Delta\boldsymbol{u} + \left(K + \frac{1}{3}\mu\right) \nabla(\nabla\cdot\boldsymbol{u}) - \gamma\nabla T_{\rm a} \tag{4}$$

and ∇ stands for the gradient operator.

In this study we solve a Danilovskaya problem for the new heat procedure, when the thermoelastic phenomenon is induced by a radiation pulse by two step action, $T_{\rm e} \to T_{\rm a} \to$ the stress, cf. [9, 10].

4. Applications

Description of laser - matter interaction may be useful in industrial applications, such as cutting, welding, material heat treatment.

In spectral analysis, arisen by the photothermal mechanism, the photoacoustic signal is useful in measuring the light absorption spectrum, particularly for transparent samples where the light absorption is very small.

Acknowledgments This work was partially supported within statutory activities No 3841/E-41/S/2017 of the Ministry of Science and Higher Education of Poland.

References

[1] S. I. Anisimov, B. L. Kapeliovich, T. L. Perelman, Electron emission from metal surfaces exposed to ultrashort laser pulses, *Sov. Phys. JETP* **39** 375–377 (1974).

[2] D. K. Ilnitsky, V. A. Khokhlov, N. A. Inogamov, V. V. Zhakhovsky, Yu. V. Petrov, K. V. Khishchenko, K. P. Migdal and S. I. Anisimov, Two-temperature hydrodynamics of laser-generated ultrashort shock waves in elasto-plastic solids *Journal of Physics: Conference Series* **500** 032021 (2014).

[3] Yu. V. Petrov, K. P. Migdal, N. A. Inogamov, V. V. Zhakhovsky, Two-temperature equation of state for aluminium and gold with electrons excited by an ultrashort laser pulse, *Applied Physics* B **119** (3) 401–411 (2015).

[4] K. P. Migdal, N. A. Inogamov, Yu. V. Petrov, V. V. Zhakhovsky, Two-temperature equations of state for d-band metals irradiated by femtosecond laser pulses, arXiv:1702.00825v3 [cond-mat.mtrl-sci] 11 Jul 2017.

[5] J. Ignaczak, The second law of thermodynamics for a two-temperature model of heat transport in metal films, *Journal of Thermal Stresses* **28** (9) 929–942 (2005).

[6] W. Bielski, P. Kowalczyk, R. Wojnar, Two-temperature heat transfer in metal films, in: *Conference Proceedings of the Numerical Heat Transfer* 2015 - Eurotherm Seminar No. 109, edited by Andrzej J. Nowak, Jerzy Banaszek, Božidar Šarler, Institute of Thermal Technology Silesian University of Technology, Institute of Heat Engineering Warsaw University of Technology, Gliwice - Warszawa 2015.

[7] W. R. Bielski, R. Wojnar, Nonstationary flow through porous piezoelectric body with two scale structure, in: *Solmech - Polish Conference of Solid Mechanics*, Kraków, 4-8 September 2006.

[8] L. D. Landau and E. M. Lifshitz, *Theory of elasticity*, Volume 7 of Course of Theoretical Physics, transl. from the Russian by J. B. Sykes and W. H. Reid, 3rd edition, Pergamon Press, Oxford - New York - Toronto - Sydney - Paris - Braunschweig 1970.

[9] V. I. Danilovskaya, Thermal stresses in an elastic semi-space due to a sudden heating of its boundary, *Prik. Mat. Mekh.* **14** (3) 316–318 (1950).

[10] A. Gałka, R. Wojnar, One-dimensional dynamic thermal stresses generated in an elastic half-space by laser pulses, *Journal of Thermal Stresses* **18** (2) 113–140 (1995).