HYSTERETIC BEHAVIOR OF RANDOM PARTICULATE COMPOSITES BY THE STOCHASTIC FINITE ELEMENT METHOD

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1. Introduction

The main purpose of this work is determination of the uncertainty level in the strain, dissipated and internal energies of particulate composite subjected to biaxial cyclic stretch. This is done for the hexagonal Representative Volume Element (RVE) of this hyper-elastic two-phase composite clearly demonstrating a hysteretic behavior. The first component – a spherical particle located centrally in the RVE – is linearly elastic and has material parameters equal to $E = 10GPa$ and $\mu = 0.3$; hyper-elastic polymeric matrix occupies 95% of the RVE. This RVE is presented in Fig. 1 and, except its initial and final configurations, it also includes the mesh. Uncertainty in this analysis results from the probabilistic constitutive model of the matrix having the given level of statistical dispersion in its stiffness resulting in different stress-strain curves. The minimum, mean and maximum values are presented in Fig. 2, where the stretching level of this RVE increases together with each of four stretch cycles and returns to zero in the end point of this analysis. Random output includes expected values, coefficients of variations, skewness and kurtosis of both elastic and dissipated energies. This analysis is a continuation of the previous considerations for an uncertainty in linear elastic particulate composites [1,2]; interested readers may refer also to Clément et al [3] or Ma et al [4], both concerning stochastic homogenization of composites.

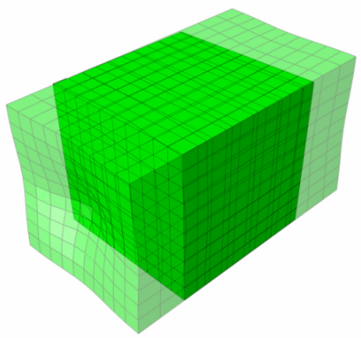

Figure 1: Discretization and deformation map of the composite RVE.

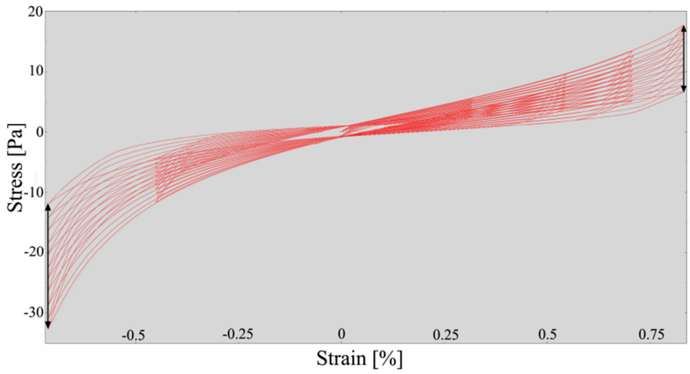

Figure 2: Hysteresis dispersion of the given composite.

2. Theory

Let us consider the unitary RVE with linearly elastic reinforcement and hyper-elastic matrix constituted below by the Van der Waals constitutive law

$$U = \mu\left\{-(\lambda_m^2 - 3)[ln(1-\eta) + \eta] - \frac{2}{3}a\left(\frac{\tilde{I}-3}{2}\right)\right\} + \frac{1}{D}\left(\frac{J_{el}^2 - 1}{2} - \ln(J_{el})\right), \quad (1)$$

where $\tilde{I} = (1-\beta)\overline{I_1} + \beta\overline{I_2}$, $\eta = \sqrt{\frac{\tilde{I}-3}{\lambda_m^2 - 3}}$ and $[I_1, I_2]$ are the first and second deviatoric strain invariants. The hysteretic behavior of this matrix is governed by the following equation:

$$\dot{\epsilon}_B^{cr} = A[\lambda_B^{cr} - 1 + E]^C(\sigma_B)^m, \quad (2)$$

where $\dot{\epsilon}_B^{cr}$ is the effective creep strain rate in network B, $\lambda_B^{cr} - 1$ is the nominal creep strain, and σ_B is the effective stress in this network. Statistical dispersion is considered for material parameters governing normal stiffness and Poisson ratio of this composite, i.e. μ and D, so that the resulting behavior of the entire composite is random and exhibits remarkable uncertainty in the stress-strain curve (see Fig. 2).

3. Results

The final results include the first four probabilistic characteristics of the maximum values of three different energies, i.e. elastic strain energy E_{el}, dissipation energy E_{cd} and the total internal energy E_i calculated for the entire RVE. This is done using three different probabilistic techniques, i.e. Iterative Stochastic Finite Element Method (ISFEM), crude Monte-Carlo simulation (MCS) and semi-analytical method (AM) with respect to the input coefficient of variation of constitutive model parameters, further denoted by c_p; these parameters are assumed to be uncorrelated and Gaussian. The expected value is presented in Fig. 3 and shows a moderate dependence on the $\alpha(c_p)$ with small decreases of all energies together with its increase. The internal energy is the largest one and is followed by the dissipated energy, while the elastic one is more than four times smaller. This is because of a cyclic stretch, where dissipated energy increases during each relaxation, while the elastic one depends on the stretch level only. The internal energy as a sum of these two is the largest during the last cycle for the ultimate strain of 0.8, where elastic strain energy is maximized. During comparison of the output coefficients of variation collected in Fig. 4 it is clearly seen that they are almost proportional and very close to the input uncertainty. The one for E_{el} is the largest, while variation of E_{cd} – exhibits minimum value; this difference increases together with an increase of $\alpha(c_p)$ and reaches approx. 30% for $\alpha(c_p) = 0.15$. Output of the three concurrent probabilistic methods shows a perfect agreement for the expected values and a little worse for the coefficients for variation, but the results are still very close to each other and within 5% tolerance.

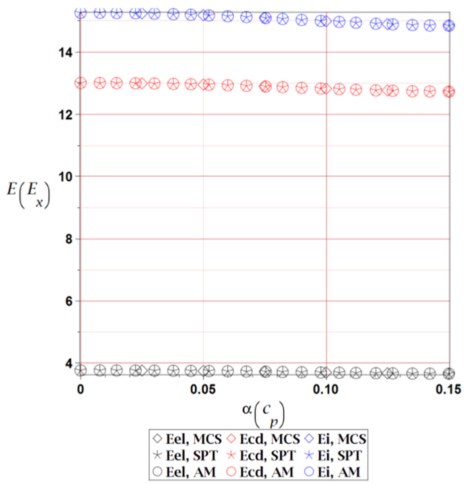

Figure 3: Expected value of the ultimate elastic strain E_{el}, dissipation E_{cd} and internal E_i energies in the RVE vs. $\alpha(c_p)$.

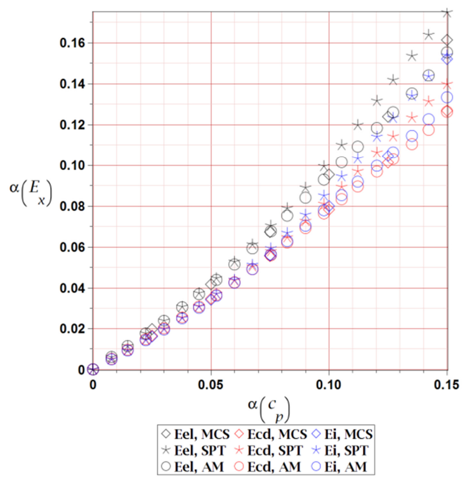

Figure 4: Coefficient of variation of ultimate elastic strain E_{el}, dissipation E_{cd} and internal E_i energies in the RVE vs. $\alpha(c_p)$.

4. Conclusions

Deformation energies in hyperelastic RVE of the composite subjected to a cyclic stretch have a certain level of uncertainty, which is almost the same as the one of the input constitutive law for the matrix. The analyzed expectations decrease a little bit together with an increase of an input uncertainty, the skewness as well as kurtosis differ both from zeros and demonstrate remarkable magnitudes.

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