STRUCTURAL OPTIMIZATION PROBLEM USING MICROTRUSS METHOD

Y. Yokotani[1] and I. Ario[1]

[1]*Hiroshima University, Kagamiyama1-4-1, Higashihiroshima, Japan*

e-mail: m185931@hiroshima-u.ac.jp

1. INTRODUCTION

The configuration of the structure is artificially determined such that the design conditions are satisfied. However, as the structure becomes larger, it is necessary to design a lightweight and robust structure. "Coat hook problem" is well-known as a bench model of such a structure optimization problem. An ideal form of Michell(1904) [1] is known as a solution to this problem. However, in this form, the necessary factor rigidity ratio and stress state of each member are not clarified. Also, there are problems of mesh dependence of solution and checkerboard phenomenon when we calculate this optimization problem using numerical methods. The purpose of this research is to clarify the optimum skeleton structure and stress state that is the minimum weight and forming the optimum morphology by discretizing the design region filled-in micro Finite Element Truss model.

2. Optimal structural form of Michell

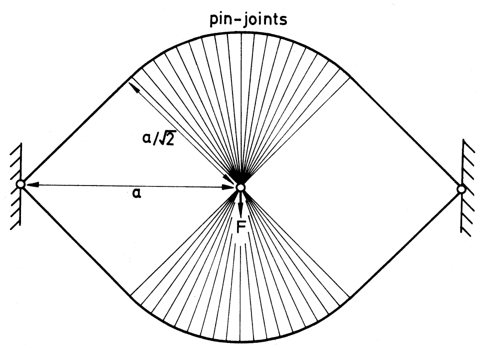


Figure 1: Concept of optimal structure of Michell(1904)

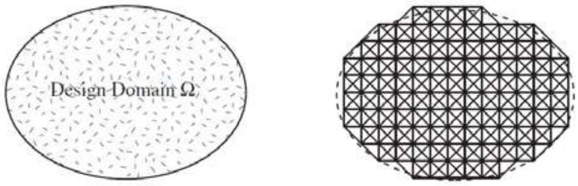
We consider the inverse analytical problem to create an efficient structural form which has given the design conditions of materials, loads and boundary. It had been suggested that the optimum structure of this problem bases on the principle stress line on his theoretic approach shown in Figure 1. This classical problem is paid attention recently to create the digital design as the model of bench mark in the field of computing mechanics. It is well−known that this problem is as the Michell's optimum truss [1].

3. OPTIMISATION STRATEGY [2]

3.1. Optimum structural form using microtruss

We represent the design domain Ω of a continuum by microtrusses $d\Omega^{(}m)$, (1). The discretisation is diagrammatically represented in Figure 2, where an example of a continuum Figure 2(a) is discretised as Figure 2(b). It is proposed that the behaviour of the domain Ω can be adequately modelled by microtrusses for large number of unit cells, m, where each microtruss member represents a pin-jointed linear extensional spring.

(1)
$$\Omega = \int d\Omega = \lim_{d\Omega \to 0} \sum_{\overline{m}}^{(1/d\Omega)\in Z} d\Omega^{(\overline{m})} \approx \sum_{\overline{m}\gg 1}^{\overline{M}} d\Omega^{(\overline{m})} \propto \sum_{m\gg 1}^{M} x^{(m)}$$



(a) Continuum, Ω (b) FEM of microtruss, $\sum \mathrm{d}\Omega$

Figure 2: Discretisation of a Continuum

3.2 . Theory of morphogenesis

The design variable is the cross-sectional area of the microtruss members, $\boldsymbol{x} \in \boldsymbol{R}^N$ and therefore the linear equilibrium equation can be rewritten as (2).

$$\boldsymbol{F}(\boldsymbol{u}, f, \boldsymbol{p}, \boldsymbol{x}) = K(\boldsymbol{x})\boldsymbol{u} - f\boldsymbol{p} = \mathbf{0} \tag{2}$$

where $\boldsymbol{u} \in \boldsymbol{R}^N$ is a displacement vector, $f \in \boldsymbol{R}$ is a load parameter and $\boldsymbol{p} \in \boldsymbol{R}^N$ is a load vector.

The goal of optimization is a minimum weight design, represented as (3) in a discrete form. Since the density and the member length are constant, a more optimum form can be formed by changing only the cross-sectional area of each member. The design modification is based on the ratio of the member stress and the average stress $\overline{\sigma}_{(\nu)}$, where the stiffness of a member is updated at each load step according to (4).

$$\begin{aligned} &\text{Minimize : } \sum_{m=1}^{M} \boldsymbol{W}^{(m)} = \sum_{m=1}^{M} \rho \boldsymbol{A}_{(\nu)}^{(m)} \ell^{(m)} \\ &\text{subject to :} \boldsymbol{F}(\boldsymbol{u}, f, \boldsymbol{p}, \boldsymbol{x}) = \mathbf{0} \\ &\qquad \boldsymbol{A}_{(\nu)}^{(m)} \leq A_{max}, \quad \sigma_{(\nu)}^{(m)} \leq \sigma_a \end{aligned} \tag{3}$$

where $\boldsymbol{W}^{(m)}$ is the weight of each member, ρ is the density, $\ell^{(m)}$ is the member length, A_{max} is the maximum cross-sectional area and σ_a is the allowable stress.

$$\boldsymbol{x}_{(\nu+1)} \;=\; \mathcal{F}(\boldsymbol{x}_{(\nu)}) = \gamma \frac{\left(\sigma_{(\nu)}^{(m)}\right)^2}{\overline{\sigma}_{(\nu)}^2} \boldsymbol{x}_{(\nu)} \tag{4}$$

where $\overline{\sigma}_{(\nu)} = \frac{1}{M}\sqrt{\sum_{m=1}^{M} \sigma_{(\nu)}^{(m)}}, \quad \nu = 1, 2, \cdots, \quad \gamma$ is optimisation rate constant.

4 . CONCLUDING REMARKS

In this research, we have constructed a presented optimization method using microtruss method and addressed the problem of structure optimization using coat hanging problem and simple beam model as an example. It is the greatest result of this presented method that the layout of the skeleton structure with the minimum weight is obtained by placing the microtruss in the design area and applying the load and the boundary condition.

References

[1] A. G. M. Michell. The limits of economy of material in framed structures, Phil. Mag. (Series 6), 8, 589-597, 1904.

[2] I. Ario, M. Nakazawa, Y. Tanaka, I. Tanikura, S. Ono. Development of a prototype deployable bridge based on origami skill, Automation in Construction, Vol.32, 104-111, 2013.