BOUNDARY ELEMENT ANALYSIS FOR UNSYMMETRIC COMPOSITE LAMINATES WITH ELASTIC INCLUSIONS

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1. Introduction

For unsymmetric composite laminates, the material properties are not symmetric with respect to the middle plane, and the deformation in stretching and bending will be coupled together. This coupling effect would then make the analysis complicated, and the methods in analyses for pure stretching or pure bending in metallic plates or symmetric laminates, are infeasible for unsymmetric laminates. Fortunately, the Stroh-like formalism has been developed to cope with the coupled deformation [1]. Since the mathematical form of Stroh-like formalism is identical to Stroh formalism for two-dimensional linear anisotropic elasticity, almost all the techniques and identities developed for the two-dimensional problems can be employed here. With this advantage, several Green's functions for unsymmetric laminates have been derived. In addition, the boundary element method (BEM) for the coupled deformation have also been successfully implemented by taking the Green's function for the infinite unsymmetric laminates as the fundamental solution [2]. With this success, in this paper we consider to develop a special boundary element by using the fundamental solution for the infinite unsymmetric laminates with elastic inclusions. Although its associated Green's function has been derived one decade ago, after numerical computation we found that some physical quantities calculated from the existing solution become discontinuous across the inclusion or on some other unexpected locations, which contradicts the assumption of continuum model and has not been noticed in the literature. Most of the discontinuities are raised by the terms containing the multi-valued complex logarithmic function [3], and some others are due to the misfit of the rigid body motion of matrix and inclusion. Additionally, in Stroh-like formalism only the values of in-plane displacements and rotation angles (with respect to x_1- or x_2- axis) are provided, the calculation of deflection requires the integral via rotation angles. Although the integration can be done analytically or numerically, to get a continuous deflection some integral constants cannot be arbitrarily neglected and should be treated carefully.

After the above proper treatment, the Green's function obtained in the literature [1] can be corrected and converted into the fundamental solution required by BEM for coupled stretching-bending analysis (which is simply named as CBEM). The analyses for unsymmetric laminates with inclusions can then be carried out, which is expected to be much more efficient and accurate than the conventional boundary element. To verify the correctness of CBEM, a symmetric laminate with elastic inclusion under in-plane loading is served as a special case whose results can be compared by BEM for two-dimensional analysis (named as 2D-BEM).

2. Numerical Examples

Consider a graphite/epoxy fiber-reinforced composite laminate with isotropic steel inclusion subjected to a uniform tensile stress $\sigma^\infty = 1$ MPa as shown in Fig. 1. To start our verification process, a symmetric laminate [45/0/0/45] is considered in the present example. The geometry of the laminate and inclusion can be represented by a/L=0.2, b/L=0.1, L=1 m, and $h_1 - h_0 = h_2 - h_1 = h_3 - h_2 = h_4 - h_3 = 1\,\mathrm{mm}$. The material properties of each lamina are $E_L = 138$ GPa, $E_T = 9$ GPa, $G_{LT} = 6.9$ GPa, $\nu_{LT} = 0.3$, whereas the properties of steel are $E = 210$ GPa, $\nu = 0.3$. Here, *E*, *G* and *v* are, respectively, Young's modulus, shear modulus, and Poisson's ratio, while the subscripts *L* and *T* represent the fiber and transverse directions. Forty linear elements and 44 nodes are used in the present example for both of CBEM and 2D-BEM. The hoop stresses σ_{ss} along the interface of the inclusion are shown in Table 1 where φ is a parameter representing the point along the inclusion boundary ($x_1 = a\cos\varphi$, $x_2 = b\sin\varphi$). The superscripts (1) and (2) denote, respectively, the values in the side of matrix and inclusion. Table 1 shows that the numerical results of CBEM are quite close to 2D-BEM analysis. Through the verification by the special case of symmetric laminate, we are now confident with the feasibility of the proposed CBEM for the general cases of coupled stretching-bending

deformation. And hence, several examples of *unsymmetric laminates* subjected to in-plane forces, bending or twisting moments, or transverse loads will then be presented in this conference.
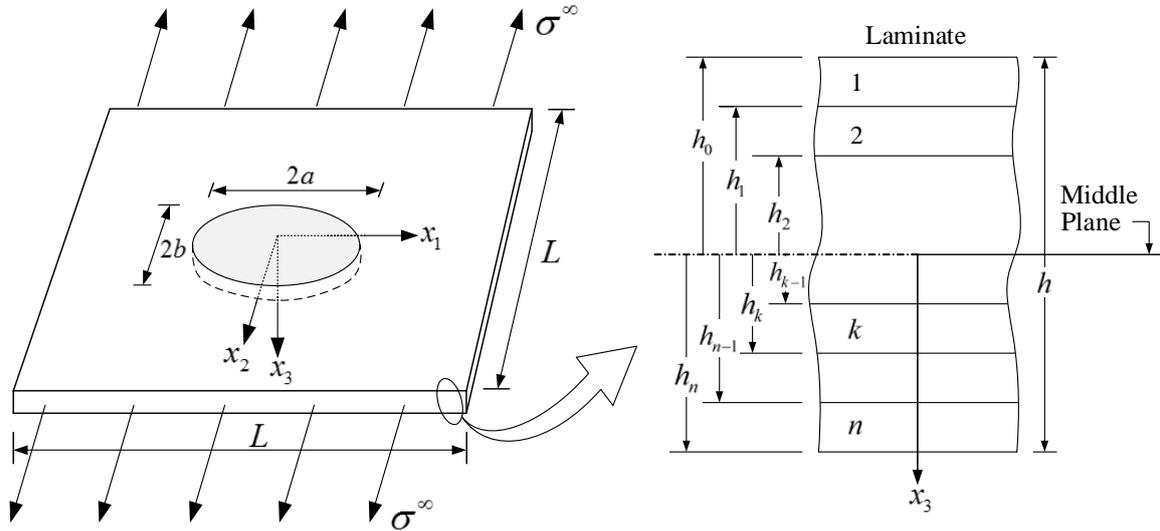
Fig. 1: A laminate with inclusion subjected to a tensile stress σ^{∞}.

φ (degree)	$\sigma_{ss}^{(1)} / \sigma^{\infty}$		$\sigma_{ss}^{(2)} / \sigma^{\infty}$	
	2D-BEM	CBEM	2D-BEM	CBEM
0	0.0145	0.0146	1.32	1.32
30	-0.409	-0.409	0.818	0.818
60	-0.805	-0.805	0.373	0.373
90	0.253	0.253	0.169	0.169
120	1.37	1.37	0.148	0.147
150	0.485	0.485	0.517	0.517
180	0.0145	0.0146	1.32	1.32

Table 1: Hoop stresses along the interface of the inclusion.

References
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[3] C. Hwu, C. L. Hsu and W. R. Chen. Corrective evaluation of multi-valued complex functions for anisotropic elasticity. *Mathematics and Mechanics of Solids,* DOI: 10.1177/1081286517728542, 2017.