A Simple and Efficient Geometric Nonlinear Rotation-Free Triangle and its Application in Drape Simulation

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The idea of the rotation-free element that possesses no rotational dofs for thin shell analyses can be dated back to the 1970s. The rotation-free element does not follow the finite element format in the sense that its integration domain is smaller than its interpolation domain. Different methods including hinge-angle, polynomial interpolation, finite volume method, subdivision surface method, smoothed finite element method, etc. have been employed to quantify the curvature and, thus, the bending energy in the integration domain. Here, a simple and efficient geometric nonlinear rotation-free triangle is presented. With reference to Figure 1, a flat corotational strain-free configuration is set up with 5^C coincident with 5' and 4^c-5^c -6^c coplanar with 4'-5' -6'. Let **n** be the unit vector normal to 4^c-5^c-6^c, the deflection from 4^c-5^c -6^c to with 4'-5'-6' parallel to **n** is

(1) $$w = \mathbf{n}^T(\mathbf{U} - \mathbf{U}^C)$$

where $\mathbf{U}^C$ is the rigid body motion that brings 1-to-6 to 1^c-to-6^c. When the radius of curvature is considerably larger than the integration domain and the inplane stretching is not significant, **n** and $\mathbf{U} - \mathbf{U}^C$ are nearly parallel. Thus,

(2) $$\mathbf{U} - \mathbf{U}^C = \mathbf{n}\mathbf{n}^T(\mathbf{U} - \mathbf{U}^C) \quad , \quad \mathbf{U},_{pq} = \mathbf{n}\mathbf{n}^T\mathbf{U},_{pq} \qquad \text{for} \qquad p, q = x, y.$$

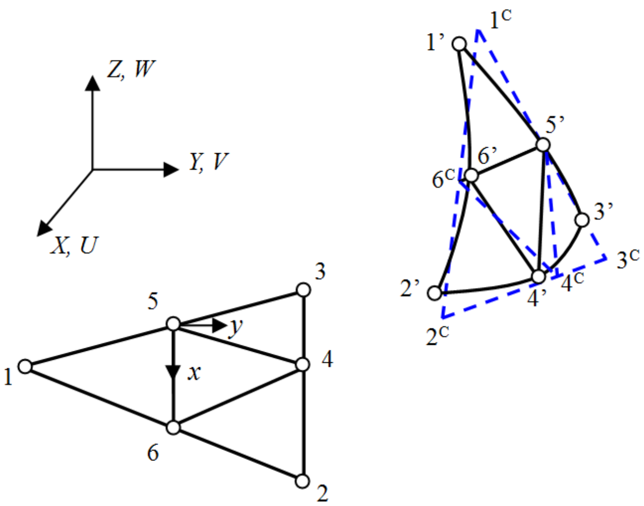
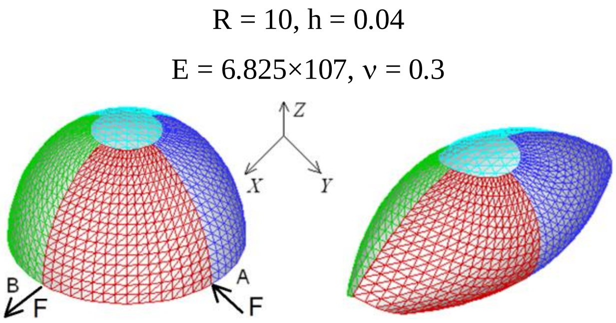


Figure 1. 1-to-6, 1'to-6' and 1^c-to-6^c show the initial, deformed and flat corotational configurations.

By virtue of the quadratic interpolation whose second order derivatives with respect to (*x*,*y*) are constant, $\mathbf{U},_{pq}$ and the displacement vector of the element patch $\mathbf{U}_{1..6}$ is related by a constant matrix **B**. The bending energy can be expressed as

(3) $$E^b = \frac{A}{2}\begin{Bmatrix} w,_{xx} \\ w,_{yy} \\ 2w,_{xy} \end{Bmatrix}^T \mathbf{D}\begin{Bmatrix} w,_{xx} \\ w,_{yy} \\ 2w,_{xy} \end{Bmatrix} = \frac{A}{2}\begin{Bmatrix} \mathbf{U},_{xx} \\ \mathbf{U},_{yy} \\ 2\mathbf{U},_{xy} \end{Bmatrix}^T \begin{bmatrix} D_{11}\mathbf{I}_3 & D_{12}\mathbf{I}_3 & D_{13}\mathbf{I}_3 \\ D_{21}\mathbf{I}_3 & D_{22}\mathbf{I}_3 & D_{23}\mathbf{I}_3 \\ D_{31}\mathbf{I}_3 & D_{32}\mathbf{I}_3 & D_{33}\mathbf{I}_3 \end{bmatrix}\begin{Bmatrix} \mathbf{U},_{xx} \\ \mathbf{U},_{yy} \\ 2\mathbf{U},_{xy} \end{Bmatrix} = \frac{A}{2}\mathbf{U}_{1..6}^T(\mathbf{B}^T\mathbf{D}\mathbf{B})\mathbf{U}_{1...6}$$

where *A* is the area of 4-5-6, $\mathbf{D} = [D_{ij}]$ is the bending rigidity matrix and **D** is self-defined. Consequently, the tangential bending stiffness matrix, which is the second derivative of E^b with respect to $\mathbf{U}_{1..6}$, is a contant matrix and needs not be updated in the iterative solution procedure. This feature renders the triangle particularly simple and efficient [1]. The membrane energy can be considered by using the CST. Figure 2 shows the prediction of the triangle in a poupluar benchmark problem. It was latter noted that a spurious folding mode appear. The mode, however, can be suppressed effectively by deriving the membrane energy from a 6-node interpolation of displacement [2], see Figures 3a and 3b.



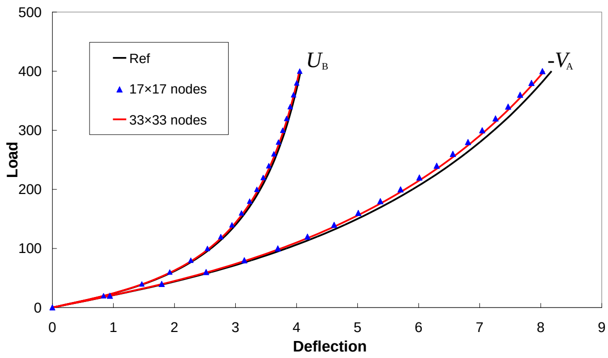
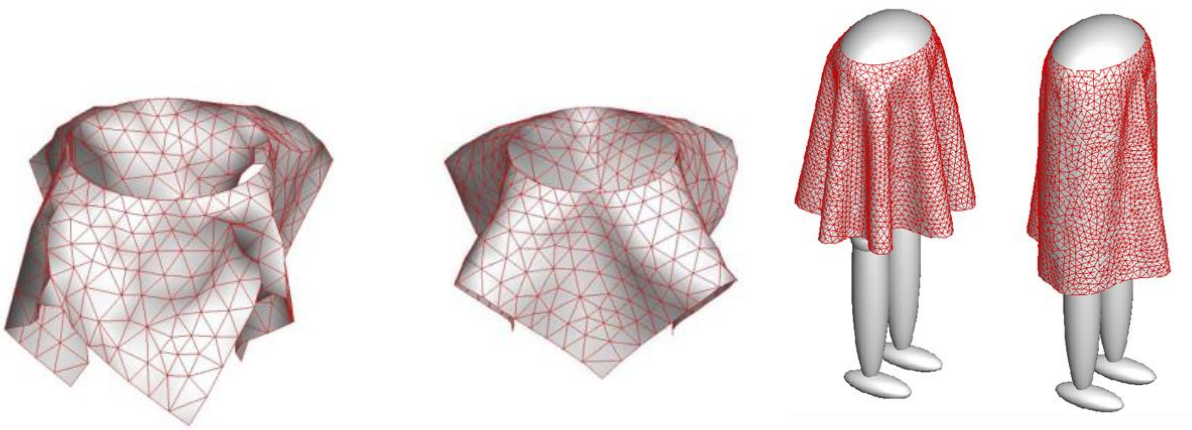


Figure 2. The undeformed and the predicted deformed geomtry in a popular elastic shell problem.

Drape simulation finds its applications in fashion design, e-commerce of clothing and production of animated movies. Fabric drapes are typical large displacement, large rotation and small strain problems. Compared with the conventional geometric non-linear shell analysis, computational drape analysis is particularly challenging due to the small bending to tensile rigidity ratio of most fabric. This presentation will discuss how the rotation-free triangle is applied to drape simulation which considers not only large displacement/rotation but also adaptive remeshing, dynamic, contact/collision and drape over moving manikin [2,3], see Figure 3c.



(a) (b) (c)

Figure 3. Non-physical sharp folds in (a) are eliminated in (b). (c) Skirts drape over manikins.

References

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